# Parallel Markets in School Choice*

Mustafa Oğuz Afacan† Piotr Evdokimov ‡ Rustamdjan Hakimov§
Bertan Turhan¶

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**Abstract**

When applying to schools, students often submit applications to distinct school systems that operate independently, which leads to waste and distortions of stability due to miscoordination. To alleviate this issue, Manjunath and Turhan (2016) introduce the Iterative Deferred Acceptance mechanism (IDA); however, this mechanism is not strategy-proof. We design an experiment to compare the performance of this mechanism under parallel markets (DecDA2) to the classic Deferred Acceptance mechanism with both divided (DecDA) and unified markets (DA). Consistent with the theory, we find that both stability and efficiency are highest under DA, intermediate under DecDA2, and lowest under DecDA. We observe that some subjects use strategic reporting when predicted, leading to improved efficiency for all participants of the market. Our findings cast doubt on whether strategy-proofness should be perceived as a universal constraint to market mechanisms.

*Keywords*: Matching markets, deferred acceptance, information acquisition, game theory, lab experiment

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†Faculty of Arts and Social Sciences, Sabancı University, 34956, Istanbul, Turkey; email: mafacan@sabanciuniv.edu

‡Higher School of Economics; email: pevdokim@gmail.com

§University of Lausanne & WZB Berlin Social Science Center; email: rustamdjan.hakimov@unil.ch

¶Iowa State University; email: bertan@iastate.edu

# 1 Introduction

In many school districts, schools are split into groups that compete for the same set of prospective students. In the US, public schools, charter schools, private schools, and voucher programs run independent admission processes, and most families participate in the admission processes of multiple school groups.[1] This decentralization is not specific to the US. In Germany, for instance, kindergartens are typically managed by independent associations with separate admission processes. In Brazil, centralized college admissions are run separately for public and private universities. In India, the two types of engineering colleges ran independent admission processes prior to 2015 (Baswana et al., 2019).

It is straightforward to see that parallel admission processes lead to inefficiencies. Some students might be assigned multiple seats while others remain unassigned. The lack of coordination between different authorities causes some school seats to be wasted.[2] To reduce the number of wasted seats, it is common practice for schools with vacant seats to have open enrollment after the main round of admissions. This is, however, generally an uncoordinated and chaotic process, where parents apply to individual schools in search of a seat for their unmatched child. This way of matching students to schools is inconvenient and does not always eliminates the waste completely. Even when it solves the issue of wasted seats, it violates students' priorities, and in turn the stability of the final allocation.

In school choice markets, match stability has become the primary concern for policymakers over the last two decades, as it implies a crucial fairness criterion.[3] To achieve a stable allocation for the overall market, the first best solution is to unify all schools under the same umbrella and use the *student-proposing deferred acceptance mechanism*

[1] We will refer to these school groups as *different authorities* that manage their own admission processes.

[2] See the related discussion in New Orleans: https://www.nola.com/news/education/article_0e350fbe-7ef9-5da6-9fd2-edeefa2af285.html (last accessed on June 22, 2020).

[3] Note that stability guarantees that no seat is wasted. In school choice settings, Balinski and Sönmez (1999) decompose stability into individual rationality, non-wastefulness, and fairness. According to the fairness criterion, a student should not be denied at a school if another student with low priority, according to the school's ranking, was offered a seat.

*(DA)*.[4] When full integration is not possible, Manjunath and Turhan (2016) propose an *iterative deferred acceptance mechanism* (IDA) that produces a stable matching for the overall market. The IDA algorithm works as follows: Each authority applies DA for its respective schools. A student who is matched to a school in more than one authority accepts her most preferred among them and turns down the rest. If there are wasted seats, each authority then computes a re-match by applying DA to the updated student preferences.[5] The re-match may still have wasted seats, so the process may be repeated. If it is repeated enough times, the resulting matching will be stable. While iterating may be costly, as it involves additional inputs from students, Manjunath and Turhan (2016) show that there are significant gains from even the first few iterations.[6]

While IDA offers a sensible solution for parallel admissions, unlike DA, it is not strategy-proof. Strategy-proofness is a highly desirable property for a school choice mechanism, as it levels the playing field (Pathak and Sönmez, 2008) and simplifies the school choice game for the participants. It was the main reason for multiple school choice reforms, where the non-strategy-proof Boston mechanism was abandoned in favor of DA. Thus, practical applicability of IDA might be hampered by its manipulability. However, the ease of recognizing and enacting manipulations may vary across manipulable mechanisms.[7] As Troyan and Morrill (2020) highlight, some mechanisms provide manipulation opportunities that are much easier for agents to identify and execute successfully than others. In their terminology, the IDA mechanism is not obviously manipulable, whereas

[4]DA was first introduced by Gale and Shapley (1962) in the context of many-to-one college admissions. It was adapted to the school choice environment by Abdulkadiroğlu and Sönmez (2003). DA produces a stable matching that is preferred to any other stable matching by every student. As a direct mechanism, it is strategy-proof. Hence, students are expected to report their true rankings over schools.

[5]If a student *accepts* a seat in an authority, her submitted preferences over schools in this authority are truncated by deleting every school that is ranked below the accepted school. If a student *rejects* a seat in a given authority, her submitted preferences over schools in this authority are truncated by deleting the school she turns down and all the schools ranked below it.

[6]When iterated only a few times, the resulting matching is individually rational and fair but might be wasteful. Each additional iteration leads to Pareto improvement. Iterating this process yields a non-wasteful matching.

[7]Pathak and Sönmez (2013); Bonkoungou and Nesterov (2020); Decerf and Van der Linden (2018), and Chen et al. (2016) introduce different criteria to compare mechanisms by their vulnerability to manipulation.

the Boston mechanism, for example, is.[8] Moreover, the relevance of the "leveling the playing field" argument depends on the mechanism and the consequences of manipulations on the placements of other students. It turns out that, under IDA, whenever strategic students play undominated strategies, they never gain at the expense of sincere students. That is, formally speaking, we show that IDA is harmless when manipulating students play undominated strategies (see Theorem A.1 in the Appendix). These considerations raise the question of whether the lack of strategy-proofness of IDA is costly in practice and invite an experimental test.

The main goal of this paper is to use laboratory experiments to compare the performance of DA run separately in each authority and the IDA procedure. Thus, we use experiments as a testbed for a new highly relevant mechanism. We consider markets with two authorities, each conducting an independent admission process. In the first treatment, each authority runs DA separately, and in case of admission to both authorities students have a choice of which school to attend (DecDA). This treatment represents the current practice of independent admissions. The second treatment implements the IDA procedure with two iterations (DecDA2): after students that are assigned a seat in both school groups choose their preferred school, their lists are truncated, and DA is run again.[9] As a baseline comparison, we also run DA on the whole market (DA), as if the school groups managed to reach an agreement of the unified admission process.

We study three markets that differ in the source of improvement of DecDA2 relative to DecDA and its potential to reach the full efficiency benchmark of DA. One source of improvement is mechanical and does not require strategic sophistication, while the

[8] Troyan and Morrill (2020)'s definition and behavioral characterization are inspired by the idea of obvious strategy-proofness (Li, 2017). Troyan and Morrill (2020) show which manipulations are obvious, classify manipulable mechanisms as either obviously manipulable or not obviously manipulable, and show that stable mechanisms are not obviously manipulable.

[9] We use DecDA2 in the experiments for the sake of speed and simplicity. We also keep in mind that, in practice, just a few iterations are sufficient for IDA to converge to a stable outcome, as is evident by the joint admission procedure of the technical universities in India (Baswana et al., 2019). The authors report that it takes three or four iterations for their version of IDA to reach a stable outcome. For the markets we used in the experiment, the Nash equilibria in undominated strategies of the game induced by DecDA2 and IDA are the same (see Remark 3 in the Appendix for details). Moreover, because IDA is (weakly) more efficient than DecDA2 at every preference profile, and this strictly holds for some problem (see Proposition A.2), DecDA2 gives a lower bound with respect to efficiency.

second depends on the ability of participants to optimally manipulate their reported preferences. The markets are as follows:

- **Market RI ("Repetition improvement")**: This market has a unique stable match due to a high correlation of school priorities. Schools in both authorities desire the same students, which leads to high inefficiency. One iteration of DA restores efficiency completely in theory. The advantage of DecDA2 is mechanical, and does not require strategic play, as DecDA2 is strategy-proof in this market.
- **Market SI ("Strategic improvement")**: This market has four stable matches, and there is no waste in DecDA due to relatively uncorrelated school priorities. There is no mechanical advantage of DecDA2 relative to DecDA. The improvement is only possible through strategic play. All students have incentives to truncate their preferences.
- **Market BOTH**: This market has two stable matches. It has both features: a mechanical advantage of DecDA2 that does not rely on strategic behavior and an additional benefit to be gained from optimal manipulation.

We run experiments between-subjects in the mechanism dimension and within-subjects on the market dimension. We test whether subjects distinguish environments from the perspective of the most relevant behavioral aspect: incentives for strategic play. Overall, our results strongly align with the theory. DecDA2 significantly improves stability and efficiency relative to DecDA in all markets. The highest improvement is observed in the market RI where it does not require strategic play. We do, however, observe significantly lower truthful reporting rates among subjects with incentives to play strategically than subjects with truthful weakly dominant strategy. This translates into higher efficiency for groups with more strategic players under DecDA2 in markets SI and BOTH, as predicted by the theory. Contrary to the predictions, however, the improvement from DecDA2 is lower than theoretically predicted, and as a result, efficiency is lower than in DA. Thus, our results are twofold: In markets when DecDA2 improves the efficiency without relying on strategic play, the improvements are the highest, while when there are incentives to manipulate, only some subjects understand them even as they still manage to improve significantly over DecDA.

Our paper provides direct and straightforward guidance for market designers who face an exogenous constraint of parallel admissions. The practical implementation of the iterated parallel DA procedure does not require much cooperation between authorities, as the admission processes remain independent to a large extent, with only slight technological integration. Additional iterations can have a high effect on the efficiency of overall allocations, and the concerns of deviations from strategy-proofness are secondary in this context. While decentralization will always lead to some efficiency loss relative to a fully centralized market, DeDA2 with iterations provides a cheap and simple second-best solution.

# 2 Literature Review

The school choice literature, starting with Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003), has assumed that there is a single clearinghouse that allocates all of the available seats. Manjunath and Turhan (2016) introduce a school choice model in which groups of schools within a single district run their admission processes independently of one another. The authors study the problem of re-matching students to take advantage of empty seats. They propose an iterative way for each school group to independently match and re-match students to its schools using DA. They show that every iteration leads to Pareto improvement and reduces waste while respecting priorities. Turhan (2019) examines the effects of partition structures of schools on students' welfare and on the incentives students face under the mechanism introduced by Manjunath and Turhan (2016).

Our paper directly relates to the experimental literature on school choice, originating with the seminal paper of Chen and Sönmez (2006). The authors show that strategy-proof mechanisms induce higher truthful reporting rates than the Boston mechanism. This finding was replicated in many subsequent experiments (see, for instance, Basteck and Mantovani (2018), Braun et al. (2014), Chen and Kesten (2015)). This empirical finding helped to convince policymakers to abandon the Boston mechanism in favor of the strategy-proof DA in some school choice districts. DA, however, might also not be strategy-proof if the list is exogenously constrained, and experimental evidence of (Calsamiglia et al., 2010) supports this prediction. Strategy-proofness, moreover, is not a guarantee of truthful behavior. Several experimental papers show that the truth-telling rates in the strategy-proof environment are sensitive to irrelevant information (Pais and Pintér (2008), Guillen and Hakimov (2017)). Even learning and strategic advice have only limited effects (Ding and Schotter (2019), Guillen and Hakimov (2018), Bó and Hakimov (2020)). These findings find support in recent empirical studies that identify suboptimal reporting in matching markets (see Hassidim et al. (2020), Rees-Jones (2017), Rees-Jones and Skowronek (2018)). Thus, strategy-proofness per se is not a guarantee of the desired allocations, which implies that empirical testing of mechanisms and the degree to which participants understand them is important to ensure the success of mechanisms in the field.

Our paper compares the strategy-proof DA to a non-strategy proof mechanism, where the manipulations are similar to the receiving side manipulations in a two-sided matching setup. The experimental literature on two-sided markets typically finds a high degree of deviation from the predicted behavior, with the proposing side manipulating despite strategy-proofness and the receiving side under-truncating (see Echenique et al. (2016), Pais et al. (2011)). Strategic manipulations by the receiving side are also the focus of Castillo and Dianat (2016), who investigate the use of truncation strategies under DA. They found that the frequencies of truncation strategies are not sensitive to the expected payoff gain, but are sensitive to the inherent risk. For more details on related experiments, we refer to a recent survey of Hakimov and Kübler (2020).

Finally, several recent papers test DA against a non-strategy proof mechanism, and the results of the comparison do not always favor DA. Klijn et al. (2019), Bó and Hakimov (2020) and Hakimov and Raghavan (2020) compare DA to a dynamic version of DA, where participants apply to schools one by one. Even though the dynamic DA is not strategy-proof, it leads to a higher rate of truthful play and stability. Cho et al. (2021) compare DA to the stable improvement cycle and the choice-augmented deferred acceptance mechanism, both of which restore efficiency under weak priorities but are not strategy-proof. The authors find no difference in truthful reporting between treatments and show that the stable improvement cycle improves efficiency over DA, as predicted. (Claudia et al., 2021) compare DA to the efficiency-adjusted deferred acceptance mechanism and find higher rates of truthful reporting in the latter, despite it being not strategy-proof.

# 3 Theoretical Analysis

There is a finite set of students $I = \{i_1, ..., i_n\}$ and a finite set of schools $S = \{s_1, ..., s_m\}$. Schools are partitioned into two sets (authorities) $S^1$ and $S^2$, i.e., $S^1 \cap S^2 = \emptyset$ and $S^1 \cup S^2 = S$. Each student $i \in I$ has strict preferences $P_i$ over $S \cup \{\emptyset\}$, where $\emptyset$ denotes remaining unmatched, i.e., the outside option. We write $P_i^k$ for student $i$'s preferences over $S^k \cup \{\emptyset\}$. Let $\mathcal{P}$ and $\mathcal{P}^k$ be the set of all strict preferences over $S \cup \{\emptyset\}$ and $S^k \cup \{\emptyset\}$, respectively. The *at-least-as-well* relation $R_i$ is obtained from $P_i$ as follows: $s\ R_i\ s^{'}$ if and only if either $s\ P_i\ s^{'}$ or $s = s^{'}$. A school $s \in S$ is **acceptable** to student $i$ if $s\ P_i\ \emptyset$; otherwise, it is **unacceptable.**

Schools have a capacity profile $q = (q_s)_{s \in S}$, where $q_s$ is the capacity of school $s$. Each school $s$ has a strict priority ordering $\succ_s$ over $S$. We write $\succ = (\succ_s)_{s \in S}$ for the schools' priority profiles. For $S' \subset S$, we write $q_{S'}$ and $\succ_{S'}$ for the capacity and priority profiles of the schools in $S'$. We refer to $(I, S, q, \succ)$ and $\left(I, S^k, q_{S^k}, \succ_{S^k}\right)$ as the **market** and the **submarket**.

A **matching** $\mu$ is an assignment of students and schools where no student receives more than one school, and no school admits more students than its capacity. For each student and school $k \in I \cup S$, we write $\mu_k$ to denote the assignment of $k$ under matching $\mu$. A matching $\mu$ is **individually rational** if, for each student $i$, $\mu_i\ R_i\ \emptyset$. A matching $\mu$ is **non-wasteful** if there is no student-school pair $(i, s)$ such that $s\ P_i\ \mu_i$ and $|\mu_s| < q_s$. Matching $\mu$ is **fair** if, for each student-school pair $(i, s)$ with $s\ P_i\ \mu_i$, $j \succ_s i$ for each $j \in \mu_s$. Matching $\mu$ is **stable** if it is individually rational, non-wasteful, and fair. A **mechanism** is a systematic procedure that assigns a matching for each problem.

## 3.1 Mechanisms and the Induced Games

We consider three mechanisms: DA, DecDA, and DecDA2. We describe each, as well as the game each induces, below.

### 3.1.1 DA

DA is a direct mechanism, and it runs as follows. Given a preference profile $P$,

**Step 1**. Each student applies to his most-preferred acceptable school. Each school, up to its quota, tentatively accepts the top priority applicants and rejects the rest.

In general,

**Step k.** Each rejected student in the previous step applies to his most-preferred acceptable school that has not rejected him. Each school, up to its quota, tentatively accepts the top priority students among the current step applicants and the tentatively accepted ones and rejects the rest.

The algorithm terminates whenever each student is tentatively accepted or has gotten a rejection from all of his acceptable schools.

DA induces a game among students where each submits a preferences $P \in \mathcal{P}$. It is well known that truthful reporting is a weakly dominant strategy for students under DA, which is a property known as **strategy-proofness.**

### 3.1.2 DecDA

DecDA is an indirect mechanism that works as follows. Each student $i$ submits a pair of preferences $(P_i^1, P_i^2) \in \mathcal{P}^1 \times \mathcal{P}^2$. Based on these submitted preferences, DA is run within each submarket $(I, S^1, q_{S^1}, \succ_{S^1})$ and $(I, S^2, q_{S^2}, \succ_{S^2})$. Note that a student may receive multiple school seats, each from different submarkets. A student receiving multiple seats rejects either of them, and the final DecDA outcome is reached.

DecDA induces a sequential-move game among students. Note that it is always optimal for students to choose their preferred schools in the last step. Therefore, students do not strategize in the school-selection stage. Hence, students only decide on their preference submissions. Moreover, because DA is used within each submarket, it is weakly dominant for students to be truthful in their preference submissions.

**Remark 1.** Because of its school-selection stage, DecDA is not a direct mechanism. However, as described above, this stage is automatic in that students always choose their preferred alternatives. Hence, it has nothing to do with the mechanism, implying that DecDA can be considered a direct mechanism, where students only report their preferences in each submarket. This, as well as the fact that the truthful reporting is a weakly dominant preference submission strategy, make DecDA strategy-proof.

### 3.1.3 DecDA2

DecDA2 is another indirect mechanism that works as follows. Same as in DecDA, each student $i$ submits a pair of preferences $(P_i^1, P_i^2) \in \mathcal{P}^1 \times \mathcal{P}^2$, and the DA outcome is calculated within each submarket. A student receiving multiple seats rejects either of them. The students' preferences are then truncated as follows: If student $i$ accepts (rejects) a school $s \in S^k$, then his preferences $P_i^k$ are truncated below $s$ (including $s$) in the sense that each school that is worse than $s$ (including $s$) becomes unacceptable while the schools' relative rankings remain he same. We then repeat DA within each submarket with respect to the truncated preferences. A student receiving multiple seats rejects either of them, and the final DecDA2 outcome is reached.

DecDA2 induces a sequential-move game. As in DecDA, it is always optimal for students to choose their preferred schools in the last step. Therefore, students do not strategize in the last school-selection stage. However, they may be strategic in school-selection right after the first DA outcome. However, we show that each student can always obtain a weakly better school by only strategizing at the preference submission stage (see Proposition A.1 and Remark 2 in the Appendix). Therefore, without loss of generality, we assume that students only decide on their preference submissions under DecDA2.

# 4 Experimental Design

In the experiment, there are four students $\{1, 2, 3, 4\}$ and four schools $\{A, B, C, D\}$, each of which has one seat. The treatments make use of three different markets and three different matching procedures, leading to a $3 \times 3$ experimental design, with the markets implemented within-subjects and the matching mechanisms implemented between-subjects. The between-subjects variation on the mechanism direction is typical for matching literature (see for instance, Chen and Sönmez (2006), Calsamiglia et al. (2010)), as explaning a mechanism requires a lot of time, and explaining more than one mechanism would be time consuming for participants. The within-subject dimension with respect to markets is driven by the practical consideration of experimental costs, and test subjects' understanding of the mechanism under different market conditions. Two of the matching mechanisms are decentralized in the sense that the set of schools is partitioned into two authorities, $\{A, B\}$ and $\{C, D\}$.

**Market "Repetition improvement"(RI):**

| School priorities: | A | B | C | D | Student preferences: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 1 | 1 | 2 | | D | C | A | C |
| | 1 | 2 | 2 | 1 | | C | A | C | A |
| | 3 | 4 | 3 | 4 | | A | D | D | B |
| | 4 | 3 | 4 | 3 | | B | B | B | D |

**Market "Strategic improvement"(SI):**

| School priorities: | A | B | C | D | Student preferences: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | | D | C | B | A |
| | 4 | 3 | 2 | 1 | | C | D | A | B |
| | 3 | 4 | 1 | 2 | | A | B | C | D |
| | 2 | 1 | 4 | 3 | | B | A | D | C |

**Market "BOTH":**

| School priorities: | A | B | C | D | Student preferences: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 1 | | D | D | B | B |
| | 3 | 1 | 1 | 3 | | B | C | D | A |
| | 4 | 3 | 2 | 4 | | A | B | C | D |
| | 2 | 4 | 4 | 2 | | C | A | A | C |

Table 1: Markets used in the experiment

A market specifies how each student $i \in \{1, 2, 3, 4\}$ ranks the four schools and how each school $j \in \{A, B, C, D\}$ ranks the four students. The three markets we use are reported in Table 1. The subjects' payoffs in the experiment are such that getting into a first-choice school pays 20 euros, getting into a second-choice school pays 15 euros, getting into a third-choice school pays 10 euros, and getting into a fourth-choice school pays 5 euros. Remaining unmatched yields a payoff of zero.

The three matching procedures we use are DA, DecDA, and DecDA2.

In **DA**, each of the four students submits a ranking of the four schools $\{A, B, C, D\}$ to a single reporting authority, which matches students to schools as follows: First, only the reported first choices are considered. Each school that receives one or more applications *provisionally* retains the highest priority student in its own priority ordering and rejects all other students. If a student is rejected, his application is sent to his next reported choice. Each school considers the new applications together with those of the provisionally retained students, and again retains only the highest student in its own priority ordering. The algorithm ends when no student can be rejected, at which point the provisional assignments are finalized.

In **DecDA**, each of the four students submits a ranking of $\{A, B\}$ to the authority in charge of $\{A, B\}$ and a ranking of $\{C, D\}$ to the authority in charge of $\{C, D\}$. The authority in charge of $\{A, B\}$ implements the DA to match students to schools $A$ and $B$, while the authority in charge of $\{C, D\}$ implements the DA to match students to schools $C$ and $D$. After both authorities announce their assignments, students who receive multiple seats—one from each authority—pick one.

**DecDA2** proceeds in the same manner as the DecDA with the exception that the assignments at the last step of DecDA are not finalized. Instead, students receive their assignments and are asked to choose among the schools they are assigned to in $\{A, B\}$ (if any) and the school they are assigned to $\{C, D\}$ (if any). After the students' choices are received, their initial reports are revised as follows: If a student accepted a school in $\{A, B\}$ ($\{C, D\}$), any school ranked lower than the accepted school is removed from her ranking in $\{A, B\}$ ($\{C, D\}$). If a student rejected a school in $\{A, B\}$ ($\{C, D\}$), the rejected school and any school ranked lower than the rejected school is removed from her ranking in $\{A, B\}$ ($\{C, D\}$). The authorities then apply the DA using the students' revised preferences. Each authority announces its assignment and students who receive multiple seats—one from each authority—pick one.

The comparison of the allocations between the three matching procedures is the main interest of the paper. The DecDA represents the typical status quo in the markets with two authorities that do not agree to unify. The DA in a unified market represents the "*first best solution,*" which is not feasible given the constraint of the separate admission processes between the authorities. We use the DA in a unified market as a benchmark. The DecDA2 is a recommended *"second best"* procedure that should theoretically improve on DecDA.

The three markets that we chose differ in terms of the incentives students face in DecDA2, the only procedure we consider that is not strategy-proof. This also means that DecDA2 has a different potential to recover losses in efficiency and stability due to decentralization. Our predictions on efficiency, stability, and individual strategies are summarized in the next subsection and discussed in more detail in the Appendix.

## 4.1 Predictions

**Stability**

We base our predictions on Nash Equilibria in undominated strategies.[10] We consider stability and the reach of the student-optimal stable match (SOSM) separately. Overall, while SOSM is always reached under DA, we predict weakly less stability and SOSM under DecDA2 and even less stability under DecDA (Table 2). We provide a brief summary of these predictions below; the technical details can be found in the Appendix.

| | Overall stability | SOSM |
|---|---|---|
| RI | DA=DecDA2>DecDA | DA=DecDA2>DecDA |
| SI | DA=DecDA2=DecDA | DA= DecDA2>DecDA |
| Both | DA≥DecDA2=DecDA | DA≥DecDA2≥DecDA |

Table 2: Predictions on stability.

In DA, the student-optimal stable match (SOSM) is reached if every student submits a weakly dominant strategy.[11] We therefore predict full stability under DA, with SOSM

[10]Note that if every student has a weakly dominant strategy, the equilibrium outcome in undominated strategies is unique.

[11]While it is a weakly dominant strategy for every student to fully reveal his preferences under DA, some of the students have other weakly dominant strategies, as we show in the Appendix.

being achieved 100% of the time.

In DecDA2, SOSM is the unique NE outcome in undominated strategies in markets RI and SI. In market BOTH, however, both stable and non-stable outcomes are achievable in equilibrium under DecDA2; we therefore predict weakly smaller levels of stability and SOSM outcomes compared to DA.

In DecDA, in market RI, student 3 remains unassigned under DecDA regardless of which combination of weakly dominant strategies is played; we therefore predict smaller levels of stability and less SOSM outcomes in DecDA than DecDA2. In market SI, the unique equilibrium outcome is the school optimal stable match (COSM) under DecDA, and thus we predict equal levels of stability and less SOSM outcomes relative to DecDA2. In market BOTH, student 4 might be unassigned under DecDA in undominated equilibria, thus, both non-stable and stable (COSM and SOSM) matches can be reached. We therefore predict similar levels of stability under DecDA compared to DecDA2, but weakly less SOSM outcomes.

**Efficiency**

The predictions on efficiency (sum of payoffs) are summarized in Table 3.

| | DA | DecDA2 | DecDA |
|---|---|---|---|
| RI | 70 | 70 | Between 40 and 50 |
| SI | 80 | 80 | 40 |
| Both | 70 | Between 40 and 70 | Between 40 and 55 |

Table 3: Predictions on efficiency.

Overall, we predict the highest levels of efficiency under DA, where SOSM is expected to be reached. Because SOSM can also be reached under DecDA2—uniquely in markets RI and SI—the SOSM sum of payoffs is always an upper bound on the predicted sum of payoffs under DecDA2. Because SOSM is never an equilibrium outcome under DecDA, the upper bound there is strictly smaller than that in DecDA2, while the lower bound is weakly smaller. Overall, the table suggests a clear efficiency ranking, with the highest levels predicted under DA, the second highest under DecDA2, and the smallest under DecDA.

**Truthfulness**

We predict that subjects are completely truthful[12] under DA and DecDA, both of which are strategy-proof. While DecDA2 is not strategy-proof in general, truthfulness is also a dominant strategy under DecDA2 in market RI; we therefore also expect subjects to be truthful in this treatment.

| | Truthfulness |
|---|---|
| RI | DA=DecDA=DecDA |
| SI | DA=DecDA>DecDA2 |
| Both | DA=DecDA≥DecDA2 |

Table 4: Predictions on truthfulness.

Under DecDA2, in market SI, no student has a weakly dominant strategy and every undominated equilibrium involves one or more students manipulating their reports. The optimal manipulations are a truncation of the reported lists above the college-optimal stable match, which is a third-ranked school of every student. We therefore predict less truthfulness under DecDA2 than under the other mechanisms in market SI. While truthful revelation can be sustained in equilibrium under DecDA2 in market BOTH, there also exist many equilibria that are not truthful. We therefore predict no more truthfulness in market BOTH under DecDA2 than under the other two mechanisms.

## 4.2 Implementation

Four sessions were run with each mechanism, with 24 subjects per session. The 24 subjects in each session were grouped into three matching groups of eight, so that the eight subjects within a matching group were randomly re-matched with each other every period (but not the subjects in the other matching groups) as the experiment proceeded. The re-matching was done to avoid reputation effects due to learning opponents' strategies. We kept smaller matching groups to increase the number of independent observations in the experiment and therefore the power of our statistical analysis.

[12] In our analysis of the data, following conventions in the experimental literature, we treat any weakly dominant strategy in DA as equivalent to truthfulness.

Each subject made decisions over the course of 27 periods. All subjects in the same session faced the same matching mechanism in all periods and were presented with different markets in the same order. This order was also fixed across mechanisms, so that for each session of DecDA there was a session each of DA and DecDA2 with the exact same sequence of markets.[13] This leads to four different order sequences in the experiment.

For each of the four order sequences, markets were fixed in three-period blocks, so that the first three periods were played with one market, the next three periods with a different market, and the last three periods with the remaining market.[14] Each order sequence therefore consisted of three 3-tuples of blocks, where the first 3-tuple represents the first nine periods, the second 3-tuple the following nine periods, and the last 3-tuple the last nine periods of the order sequence. The allocation of the three markets across each 3-tuple of blocks was randomly determined. We randomized at the 3-tuple (as opposed to 9-tuple) level to allow for learning effects within markets, and simplified the environment for participants. The order sequences used in the experiment are reported in Table 5.

| Order | Sessions |
|---|---|
| SI, RI, BOTH, RI, BOTH, SI, RI, SI, BOTH | Sess. 1 (DecDA), Sess. 2 (DecDA2), Sess. 5 (DA) |
| RI, SI, BOTH, RI, SI, BOTH, SI, RI, BOTH | Sess. 3 (DecDA), Sess. 4 (DecDA2), Sess. 8 (DA) |
| BOTH, SI, RI, RI, BOTH, SI, SI, RI, BOTH | Sess. 6 (DecDA), Sess. 7 (DecDA2), Sess. 9 (DA) |
| BOTH, SI, RI, BOTH, SI, RI, RI, BOTH, SI | Sess. 10 (DecDA), Sess. 11 (DecDA2), Sess. 12 (DA) |

Table 5: Market order. Each number in the left column represents a three-period block. "Sess." stands for session. For instance, in Session 1, Market SI was played in periods 1-3, Market RI in periods 4-6, Market BOTH in periods 7-9, and so on.

The participants knew the preferences of all other participants and the priorities of schools in each round. The complete information setup is the most favorable for the manipulations to occur, and thus presents an extreme test for subjects' ability to manipulate under DecDA2.

[13]This is to keep possible order effects the same across treatments.

[14]Different markets have different strategic properties, and we designed the block order to allow for learning within the market.

# 5 Results

Our focus is on testing whether DecDA2 leads to improved allocations relative to DecDA, using the results of DA (in which there are no constraints of decentralization) as the benchmark limit for improvement. We first present the across-market results on efficiency and stability. Next, we shed light on these results by presenting subjects' strategies, with a focus on truth-telling and equilibrium behavior.

For all tests, we report the p-values of the coefficient on the treatment dummy in OLS regressions of the dependent variable of interest. Standard errors are clustered at the level of matching groups, and the sample is restricted to the treatments of interest for the test. We present results for the pooled data with different market orderings and control for order effects in robustness checks. None of our main results are affected by the market order.

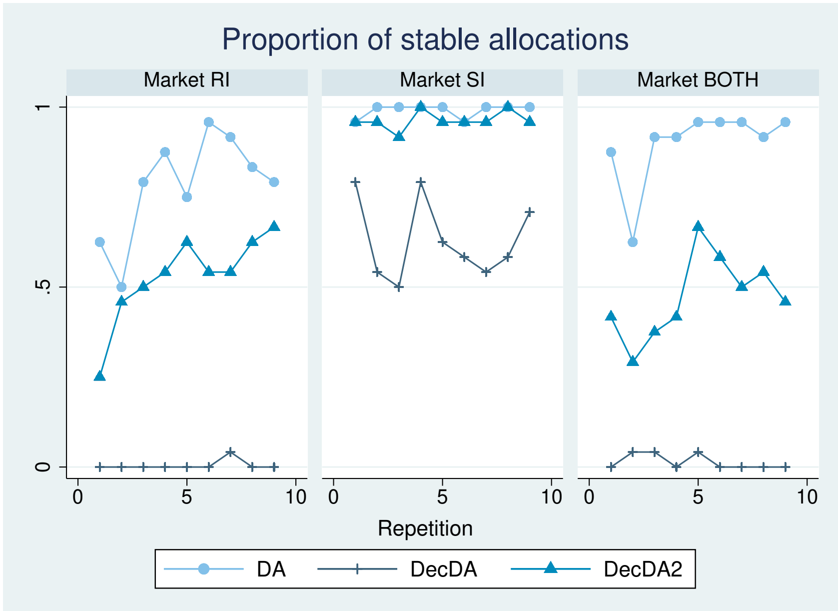
## 5.1 Stability

Figure 1a presents the proportion of stable allocations in each treatment (i.e., market and allocation mechanism). Overall, we find a clear ranking of the matching procedures, with DA leading to the highest stability, DecDA to the lowest, and DecDA2 falling in between. All of the pairwise differences are significant with $P < 0.01$, with the exception of the difference between DecDA2 and DA in market SI ($P = 0.03$).

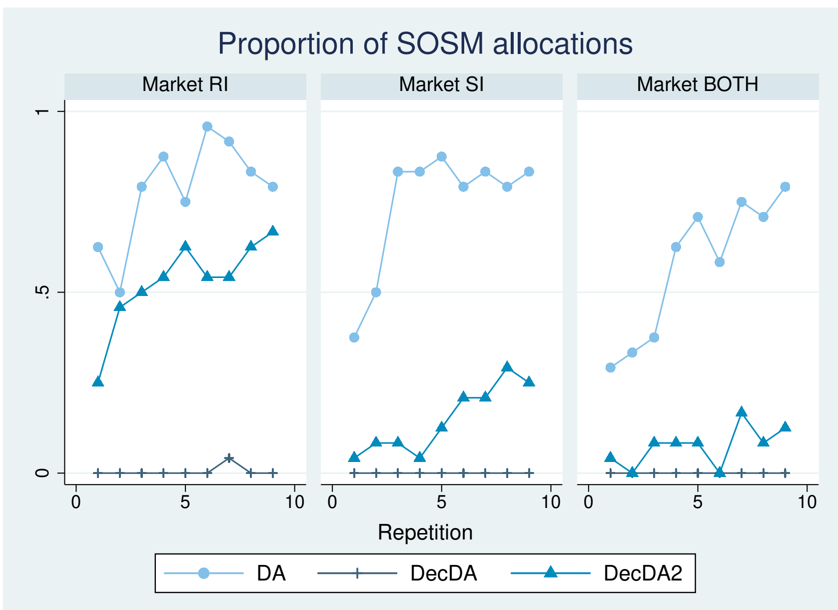
While the empirical relation of stability rankings is clear and does not depend on the market, note that theory predicted different relations, depending on the market. Two major deviations from the theory are:

1. While DecDA2 was predicted to reach full stability in markets RI and SI, realized stability levels are lower under DecDA2 than under DA. Note that misrepresentation of reported preferences is not required to reach full stability under DecDA2 in these markets.

2. In market SI, we predicted full stability under DecDA, but realized stability is lower than under DA and DecDA2. Note that the prediction is based on participants playing dominant strategies in DecDA, just like in DA.




(a) Overall stability.




(b) SOSM allocations.

Figure 1: Proportion of SOSM allocations by treatment.

Figure 1b presents the proportion of student-optimal stable match (SOSM) allocations in each treatment. Again, we observe a clear ranking of the matching procedures, with DA leading to the highest proportion of SOSM, DecDA2 the second highest, and DecDA the lowest. All of the pairwise differences are significant with $P < 0.01$. While SOSM is essentially never reached under DecDA, as predicted, the level of SOSM under DecDA2 is lower than predicted in markets RI and SI. Note that in market SI, the prediction of equal levels of SOSM under DA and DecDA2 is based on the strategic play of participants in DecDA2, and we observe a significant deviation from this prediction. Note also that the difference is larger than in market RI ($P < 0.01$), where the prediction of an equal level of SOSM is based on truthful reporting in both DA and DecDA2. These results suggest that participants only partially use strategic play.

We summarize our results as follows:

**Result 1 (Stability):** The proportion of stable allocations and student-optimal stable matches is the highest in DA, the second highest in DecDA2, and the lowest in DecDA in all markets, with all differences being significant.

## 5.2 Efficiency

Figure 2 presents the normalized efficiency levels by treatments. Normalized efficiency is calculated by dividing the realized sum of payoffs of the group by the sum of payoffs in the in the student-optimal stable match. In all markets, there is a clear efficiency ranking, with all differences statistically significant with $P < 0.01$. DecDA leads to the lowest efficiency in all markets, DA leads to the highest efficiency, and DecDA2 attains an intermediate level.

Of primary interest is the extent to which efficiency is restored in DecDA2 relative to DecDA. While DecDA2 leads to significantly higher efficiency than DecDA, the improvement is not substantial enough to reach the DA level and completely overcome the inefficiency of dividing the market into separate authorities. The normalized efficiency levels of DecDA2 and DA are the closest in market RI, where the improvement results from the elimination of empty seats and does not require strategic play from the partic-

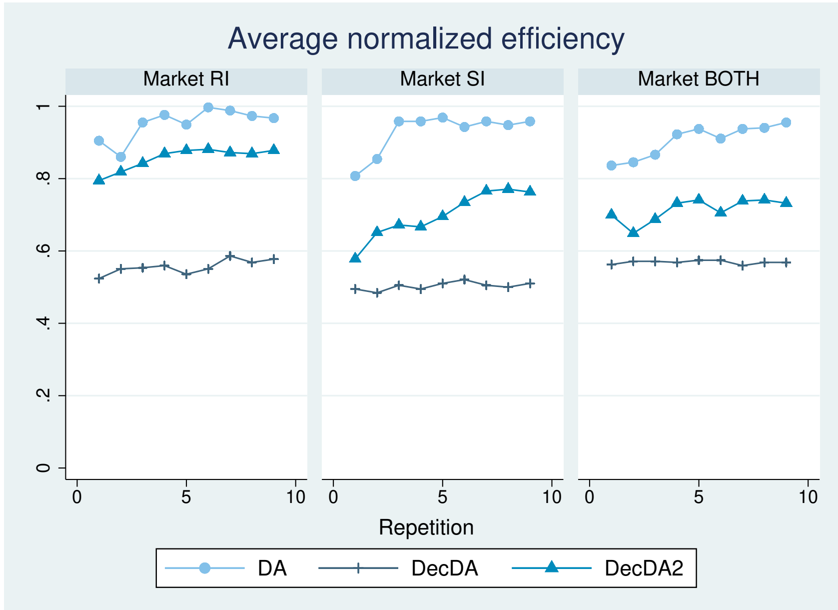



Figure 2: Average efficiency (normalized by the SOSM level) by treatments.

ipants. The improvement of DecDA2 over DecDA is smaller, but still highly significant in markets SI and BOTH, where it requires strategic play. We highlight our results as follows:

**Result 2 (Efficiency):** Efficiency is the highest in DA, the second highest in DecDA2, and the lowest in DecDA in all markets, with all differences being significant.

Thus, using stability and efficiency as evaluation criteria produces similar mechanism rankings. While DA implemented in the centralized market is the best alternative, in the presence of a constraint for having different authorities, DecDA2 leads to significant improvements over DecDA in all markets we consider. Note that these improvements put different demands on strategic behavior, depending on the market. In the next section, we analyze subjects' individual strategies in order to shed light on the main treatment effects.

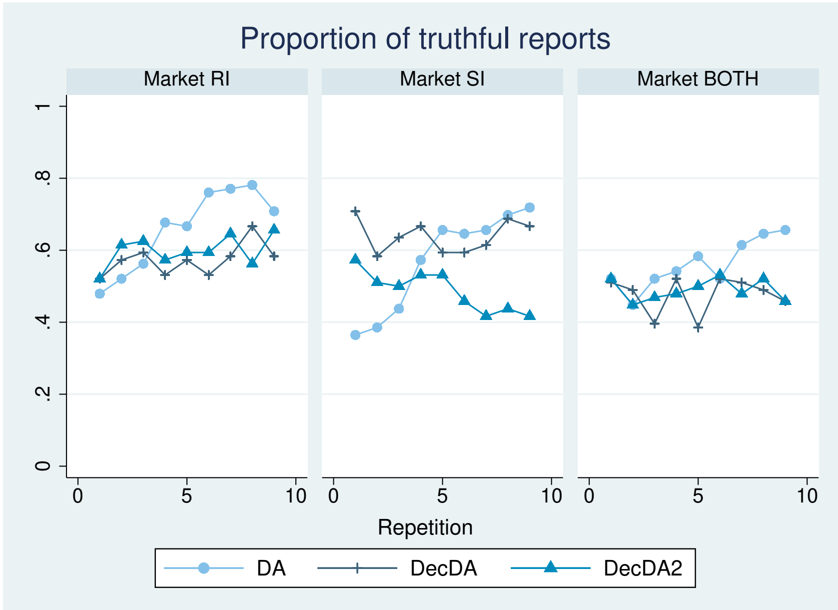



Figure 3: Truthfulness.

### 5.3 Individual Behavior

Figure 3 shows the percentages of truthful strategies in all treatments.[15] Recall that we predicted reports to be fully truthful under DA and DecDA in all markets, as well as under DecDA2 in market RI. Overall, subjects are far from fully truthful when this is predicted. On average, the participants report truthfully in 59.7% of rounds in DA, with 65.3% in the last block. This is in line with typical rates of truthful reporting documented in the literature (see Hakimov and Kübler (2020)).

In market RI, where truthful reporting is predicted for all three procedures, the proportion of truthful reports is significantly higher under DA than under DecDA ($p < 0.01$) and DecDA2 (p=0.05), with no significant differences between DecDA and DecDA2. In market SI, the proportion of truthful strategies under DA and DecDA is significantly higher than under DecDA2 ($p < 0.01$), which is in line with our predictions. In market

[15] We define a strategy as truthful if the report completely reveals the student's preferences or represents a dominant strategy, and the choice of school in case of assignments in both of the authorities corresponds to true preferences. Note that only 38 out of 2,749 choices between schools assigned in different authorities violated the participants' true preferences.

BOTH, the highest proportion of truthful reports is under DA, which is significantly higher than under DecDA (p=0.014) and weakly higher than under DecDA2 (p=0.052). There is no significant difference in the average proportions of truthful reports between DecDA and DecDA2 (p=0.82) in market BOTH.

Overall, the results only partially align with our predictions. While we observe a relatively low rate of truthful strategies, in a complete information setup with repeated play subjects might adapt equilibrium strategies that are dominated but nevertheless lead to high stability and efficiency. However, the question we focus on is whether non-strategy-proofness of DecDA2 leads to a lower rate of truthful reporting and whether participants understand the incentives to truncate their reported lists. We take an explorative approach to studying the propensity to play truthfully as follows.

We construct two variables that might influence participants' propensity to report truthfully:

1. The variable "Incentives to lie" is a dummy variable equal to 1 if the student has incentives to manipulate her reports in a market. More precisely, the dummy is equal to 1 if the student misreports her preferences in one of the undominated equilibrium strategies. This is true for all students in market SI and for students 2 and 3 in market BOTH under DecDA2. The variable aims to disentangle the effect of DecDA2 as a mechanism from the effect of incentives to manipulate the reports under DecDA2.

2. The variable "Unassigned in equilibrium" is a dummy variable equal to 1 if the subject is not assigned in at least one of the market's equilibrium outcomes. These are students 3 and 4 in market RI in DecDA and student 4 in market BOTH under DecDA and DecDA2. The idea is that being unassigned might push subjects to experiment with reports, as they have nothing to lose.

Table 6 reports marginal effects of probit regressions where the probability of playing a truthful strategy is modeled as a function of several explanatory variables.

Model (1) presents the results for the full sample. Variable "Incentives to lie" has a negative and significant effect on the probability of truthful reporting, showing that subjects indeed understand that truncation might be profitable for them. Notably, the

| | Truthful All (1) | Truthful First block (2) | Truthful Second block (3) | Truthful Third block (4) |
|---|---|---|---|---|
| DecDA | 0.03 | 0.15*** | -0.02 | -0.04 |
| | (0.02) | (0.03) | (0.03) | (0.03) |
| DecDA2 | 0.06** | 0.16*** | 0.04 | -0.03 |
| | (0.03) | (0.03) | (0.03) | (0.04) |
| Market SI | -0.01 | -0.03 | 0.02 | -0.02 |
| | (0.03) | (0.04) | (0.04) | (0.03) |
| Market BOTH | -0.07** | -0.05 | -0.06** | -0.08** |
| | (0.03) | (0.04) | (0.03) | (0.03) |
| Order 2 | -0.00 | 0.00 | 0.01 | -0.01 |
| | (0.03) | (0.03) | (0.04) | (0.04) |
| Order 3 | -0.01 | 0.00 | -0.01 | -0.02 |
| | (0.02) | (0.02) | (0.03) | (0.03) |
| Order 4 | 0.01 | 0.03 | 0.00 | -0.01 |
| | (0.03) | (0.03) | (0.04) | (0.03) |
| Incentives to lie | -0.22*** | -0.16*** | -0.24*** | -0.26*** |
| | (0.02) | (0.04) | (0.03) | (0.02) |
| Unassigned in equilibrium | -0.26*** | -0.30*** | -0.21*** | -0.26*** |
| | (0.02) | (0.02) | (0.04) | (0.03) |
| Observations | 7776 | 2592 | 2592 | 2592 |

Table 6: Determinants of truthful reporting

Notes: Marginal effects of a probit model on truthful reporting for the whole sample, and each block of repetitions separately. DecDA and DecDA2 are dummies for the corresponding treatments. Market SI and Market BOTH are dummies for the corresponding markets. Orders 2 to 4 are dummies for orders of markets, corresponding to orders in Table 5. Values in parentheses represent standard errors. Standard errors are clustered at the level of matching groups. $*p < 0.1, **p < 0.05, ***p < 0.01$.

effect is robust for each repetition block (models (2)-(4)), and the effect is increasing in magnitude with experience.

Subjects who are unassigned in at least one equilibrium of a market are significantly more likely to deviate from truthful reporting, independent of incentives. While this is a deviation from predicted behavior, it would often not be a payoff relevant deviation, as subjects are unassigned. The effect of the “Unassigned in equilibrium” variable is relatively stable across repetitions.

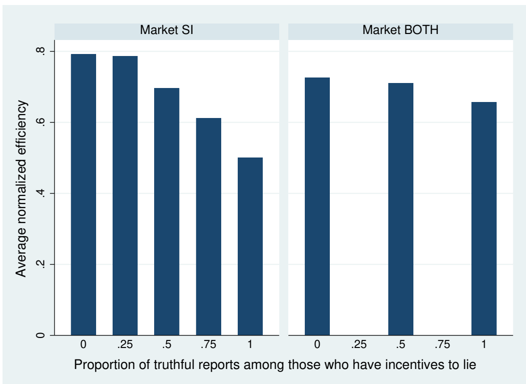
Market BOTH leads to a significantly lower rate of truthful reporting, especially with experience. The effect is small and might be driven by the presence of several equilibrium outcomes in undominated strategies in DecDA and DecDA2 (2 and 3, respectively).

The order of the markets does not have significant effects on truthful reporting.

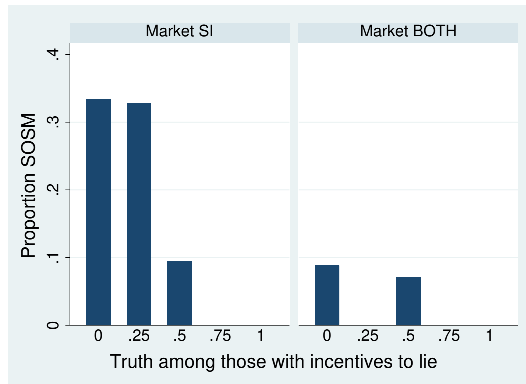
When the entire sample is considered, DecDA2 has a weakly higher percentage of truthful reporting than DA, on average, controlling for incentives to manipulate the reports. However, the effect is driven by the first block of repetition. In the first block (model (2)), both DecDA and DecDA2 have a higher rate of truthful reporting than DA, which is likely driven by the shorter length of the reported lists.[16] Note that with experience, the significance of the treatment dummies vanishes, and the differences in truthful reports between all three procedures are captured by other explanatory variables. We conclude that the low rate of truthful reports is explained by misreporting from those who have incentives to truncate their reports. Moreover, students who are at risk of being unassigned misreport their preferences despite having a weakly dominant strategy of truthful reporting, which is in line with a low probability of payoff-relevant manipulations.

We explore the relationship between individual behavior and outcomes as follows. First, we focus on markets under DecDA2 where one or more subjects has an incentive to lie, namely markets SI and BOTH. Note that all four subjects have an incentive to lie in market SI, while only two subjects have an incentive to lie in market BOTH. Second, for each matched group in these markets, we count the number of subjects with

[16] It has been reported in the literature that the longer the list that has to be submitted, the more likely subjects are to misrepresent their preferences, especially in a one-shot, i.e., inexperienced, environment (see the overview in Hakimov and Kübler (2020).




(a) Efficiency.




(b) SOSM allocations.

Figure 4: The relationship between individual behavior and outcomes.

| | (1)<br>Efficiency | (2)<br>SOSM |
|---|---|---|
| Proportion of truthful among those with incentives to lie in market SI | -0.34*** | -0.55*** |
| | (0.06) | (0.15) |
| Proportion of truthful among those with incentives to lie in market BOTH | -0.05 *** | -0.06 |
| | (0.01) | (0.05) |
| Observations | 432 | 432 |

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: The effect of truth-telling on outcomes when incentives to lie are present on outcomes.

an incentive to lie that lied. Third, we define a variable TAL (truth among those with incentives to lie) that captures the fraction of subjects in a group with an incentive to lie that reported truthfully. Plotting the data for markets SI and BOTH separately, Figure 7 shows the relationship between TAL and efficiency (a) and the probability of reaching SOSM (b). Note that if subjects misreport optimally, in market SI, SOSM is the unique equilibrium outcome, and thus efficiency is predicted to be 100%. As for market BOTH, SOSM is only one out of three equilibrium outcomes, and efficiency in an equilibrium can be as low as 57%. As reported in Table 7, TAL has a negative and significant effect on efficiency in both markets SI ($P < 0.01$) and BOTH ($P < 0.05$), in line with predictions, as strategic play is a source of efficiency gain over DecDA in these markets. The relationship between TAL and SOSM is negative in market SI ($P < 0.01$) and insignificant in market BOTH ($P = 0.25$), again highlighting the difficulty of finding optimal strategies in market BOTH. Overall, these results align with our intuition that truth-telling leads to worse outcomes for the group in treatments where incentives to lie are present.

# 6 Conclusion

We present the first experimental investigation of the IDA mechanism in partitioned matching markets. We investigate the performance of this mechanism and whether the absence of strategy-proofness causes preference misreportings and worse outcome. Our primary focus is efficiency, as partitioned markets often lead to waste. The experimental results show that efficiency is significantly higher under DecDA2 than under DecDA. Moreover, the improvement is not only driven by mechanical iterations, but also by the strategic behavior of students when this is optimal, which leads to stable matches closer to the student-optimal one. The levels are, however, lower than in centralized DA. Our results strongly support switching to IDA for markets, where assignments are currently split, as the benefits strongly outweight the costs of losing strategy-proofness.

One might argue that while improved efficiency is encouraging, the lack of strategy-proofness may hurt naive players, thus raising fairness concerns. While the "leveling the playing field" argument is important, it is not a salient concern for IDA, because whenever sophisticated students play undominated strategies, they never gain at the expense of sincere students. We not only prove this result theoretically in the Appendix, but also find empirical support for it, as efficiency increases with the higher proportion of strategic players under DecDA2 in markets SI and BOTH.

While centralized DA would be an ideal solution, centralization is not always possible. Our solution provides a fast partial fix in partitioned markets, which is feasible and does not require much cooperation from different authorities. The cost of the procedure is lack of strategy-proofness, and that it lasts for several rounds, with an additional decision being made by students between rounds. However, it still presents a significant improvement over the unstructured and often ad-hoc post-allocation scramble.

With a slightly higher effort of cooperation between the authorities, one can make the procedure even faster if the students are asked to submit a full rank order list. Then, a computer splits the list and sends the corresponding lists to each of the authorities. Once first round assignments are completed, students' decisions regarding multiple assignments are made based on the submitted list, and the procedure is re-run automatically. The joint seat allocation of Indian technical universities is implemented via this version, and it needs three or four iterations to reach a stable outcome according to data (Baswana et al., 2019).

## Appendix

# A Theoretical Results

Let us write $\psi$ for the Iterative Deferred Acceptance (IDA).[17] If we write $\sigma_i$ for a *strategy* of a student $i$, then $\sigma_i = (P_i^1, P_i^2, C_i^1, .., C_i^t)$, where for each $\ell \in \{1, .., t\}$, $C_i^\ell$ is a choice function that maps each subset of schools to a school from this set. Note that here $t$ is fixed, and it is equal to the maximum number of iterations that can occur (it is bounded from above). Let $\sigma = (\sigma_i)_{i\in N}$ be a *strategy profile.* For $k \in \{1,2\}$ and $\ell \in \{1, .., t\}$, we write $\sigma_i^k(P) = P_i^k$ and $\sigma_i^\ell(C) = C_i^\ell$.

We say that $C_i^\ell$ *agrees with* $P_i$ if, for each $S' \subseteq S$, $C_i^\ell(S')$ is the most preferred school among those in $S'$. For each student $i$ and submarket $S^k$, $P_i^k$ agrees with $P_i$ if the restriction of $P_i$ to $S^k$ is the same as $P_i^k$. A strategy $\sigma_i$ is *sincere* if, for each $k \in \{1,2\}$ and $\ell \in \{1, .., t\}$, both $\sigma_i^k(P)$ and $\sigma_i^\ell(C)$ agree with $P_i$.

Proposition A.1. *Let $P$ be a problem and $\sigma$ be a strategy profile. For each agent $i$, there exists a strategy $\sigma_i'$ such that for each $\ell \in \{1, .., t\}$, $\sigma_i'^\ell(C)$ agrees with $P_i$ and $\psi_i(\sigma_i', \sigma_{-i})\ R_i\ \psi_i(\sigma)$.*

*Proof.* Let $\psi_i(\sigma) = a$. Suppose that agent $i$'s preferences are truncated for the last time at Step $m$ of $\psi$. That is, after this step, his preferences are never truncated in $\psi$ at $\sigma$. Let $\sigma_i'$ be such that for each $k \in \{1,2\}$, $\sigma_i'^k(P)$ is the same as these last truncated preferences in submarket $k$ under $\sigma_i$.

As $DA$ is used in each submarket iteratively, no student $j \neq i$ is worse off at $\psi(\sigma_i', \sigma_{-i})$. This as well as the definition of $\sigma_i'$ implies that student $i$ receives at least a weakly better school at $\psi(\sigma_i', \sigma_{-i})$, finishing the proof. □

Because of Proposition A.1, we only consider strategies whose choices agree with the preferences. Therefore, we suppress the dependency of $\psi$ on the choices from its notation.

[17] We do not include its formal definition, as it works the same as $DecDA2$ except that it keeps iterating $DA$ until no student rejects an offer. Please see the Introduction for the verbal definition.

**Remark 2.** Note that Proposition A.1 is independent of the maximum number of iterations, shown by $t$ above. Therefore, if we set $t = 2$, then we obtain the same result for $DecDA2$.

The following definitions are from Afacan and Dur (2017).

Definition A.1.
*i. A mechanism $\psi$ is harmful at problem $P$ if there exists a pair of agents $i, j$ and $P'_i$ such that $\psi_i(P'_i, P_{-i})\ P_i\ \psi_i(P)$ and $\psi_j(P)\ P_j\ \psi_j(P'_i, P_{-i})$.*
*ii. A mechanism $\psi$ is weakly harmful at problem $P$ if there exists a pair of agents $i, j$ and $P'_i$ such that $\psi_i(P'_i, P_{-i})\ R_i\ \psi_i(P)$ and $\psi_j(P)\ P_j\ \psi_j(P'_i, P_{-i})$.*

A mechanism is *weakly harmless* if it is never harmful. A mechanism is *harmless* if it is never weakly harmful.

For a preferences $P_i^k$ and school $s \in S^k$, let us write $U(P_i^k, s) = \{s' \in S^k :\ s'\ R_i\ s\}$. In what follows, for ease of notation, we write $P_i = (P_i^1, P_i^2)$ for each student $i$. Let us also write $P = (P_i)_{i \in I}$.

Lemma A.1. *Suppose for a problem $P$, agent $i$, and $P'_i$, we have $\psi_i(P'_i, P_{-i})\ R_i\ \psi_i(P)$. If, for some student $j$, $\psi_j(P)\ P_j\ \psi_j(P')$, then there exists some school $s$ such that it becomes unacceptable under $\psi$ at $P$ for agent $i$, but not under $P'$.*

*Proof.* Let $\psi_i(P') = a$ and $\psi_i(P) = b$. By our supposition, we have $a\ R_i\ b$. Let Step $m$ be the last step of $\psi$ at $P$ where agent $i$'s preferences is truncated (this means that it is never truncated after that). Let us consider the set of acceptable schools of student $i$ by the end of Step $m$ at $P$. If there is no school that becomes unacceptable in $\psi$ at $P$, but not at $P'$, then the competition under $P'$ ultimately becomes at least weakly less fierce than under $P$. This, in turn, implies that no agent is worse off at $P'$, contradicting our supposition. $\square$

A strategy $P'_i$ is *dominated* if, for some $P''_i$, $\psi_i(P''_i, P_{-i})\ R_i\ \psi_i(P'_i, P_{-i})$ for each $P_{-i}$, where it strictly holds for some $P_{-i}$. A strategy is *undominated* if it is not dominated. A mechanism $\psi$ is *harmless in undominated strategies* if, at each problem $P$, agent $i$, and undominated strategy $P'_i$ with $\psi_i(P'_i, P_{-i})\ R_i\ \psi_i(P)$, there is no agent $j$ with $\psi_j(P)\ P_j\ \psi_j(P'_i, P_{-i})$.

Theorem A.1. *$\psi$ is harmless in undominated strategies.*

*Proof.* Let $P$ be a problem and $P' = (P'_i, P_{-i})$ where $\psi_i(P')\ R_i\ \psi_i(P)$. Suppose for some student $j$, $\psi_j(P)\ P_j\ \psi_j(P')$. By Lemma $A.1$, there exists some school $s$ such that it becomes unacceptable under $\psi$ at $P$, while it does not under $P'$. This implies that $\psi_i(P)\ P_i\ s$ yet $s\ P'_i\ \psi_i(P')$. Note that as we focus on preference misreportings (we consider sincere choices), all $s, \psi_i(P)$, and $\psi_i(P')$ are in the same submarket.

Let us now consider $P''_i$ where the set of acceptable schools are the same as those under $P'_i$, with the same relative rankings except $\psi_i(P)\ P''_i\ s$. At $P'$, as student $i$ does not obtain school $s$ anyhow, the competition does not increase under $P''$, meaning that student $i$ does not lose by reporting $P''_i$ at $P$. Moreover, as $DA$ is used in each submarket, student $i$ never gains by swapping $\psi_i(P)$ and $s$. On the other hand, for some preference submissions by the other students, he may obtain school $s$ under $P'_i$ whereas he obtains a better school under $P''_i$, (for instance, he may obtain $\psi_i(P)$ at $P''_i$ while $s$ under $P'_i$). All these show that $P'_i$ is dominated, finishing the proof. □

Proposition A.2. *For each $P$ and student $i$, $\psi_i(P)\ R_i\ DecDA2_i(P)$, and it holds strictly for some student at some problem.*

*Proof.* $\psi$ works as the same $DecDA2$ until the latter reaches its outcome. Therefore, whenever $DecDA2$ terminates, each student already receives the same school under $\psi$. Moreover, $\psi$ continues its $DA$ iterations unless no student rejects an offer. No student is worse off in the later iterations, showing that each student at least weakly prefers his school under $\psi$ to that under $DecDA2$. That is, for each $P$ and student $i$, $\psi_i(P)\ R_i\ DecDA2_i(P)$.

Let us consider a problem where $I = \{i, j, k\}$ and $S^1 = \{a, b\}$ and $S^2 = \{c\}$, each with a unit quota. Suppose that the priorities are the same at all schools in the order of $i, j, j$. Let the preferences be such that $P_i : a, c, \emptyset$, $P_j :\ c, b, \emptyset$, and $P_k :\ b, \emptyset$.

Let $\mu$ be the $DecDA2$ outcome, and it is such that $\mu_i = a$, $\mu_j = c$, and $\mu_k = \emptyset$. On the other hand, if we write $\mu'$ for the $\psi$ outcome, then $\mu'_i = a$, $\mu'_j = c$, and $\mu'_k = b$, completing the proof. □

# B Predictions

## B.1 DA

**Weakly dominant strategies in market RI**

- Student 1: {DCAB,DCBA,DCA,DCB,DC}
- Student 2: {CADB,CABD,CAD,CAB,CA}
- Student 3:{ACBD}
- Student 4: {CABD}

The unique outcome the weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & A & B \end{pmatrix}$, which is the student-optimal stable outcome.

**Weakly dominant strategies in market SI**

- Student 1: {DCAB,DCA}
- Student 2 {CDBA,CDB}
- Student 3: {BACD,BAC}
- Student 4: {ABDC,ABD}

The unique outcome of the weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & B & A \end{pmatrix}$, which is the student-optimal stable outcome.

**Weakly dominant strategies in market BOTH**

- Student 1: {DABC, DACB, DBAC, DBCA, DCAB, DCBA, DAB, DBA, DAC, DCA, DBC, DCB, DA, DB, DC, D}
- Student 2: {DCBA, DCB}

- Student 3: {BDCA, BDC}
- Student 4: {BADC}

The unique outcome under weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & B & A \end{pmatrix}$.

## B.2 DecDA

**Weakly dominant strategies in market RI**

- Student 1: $\{(AB, DC), (A, DC), (BA, DC), (B, DC), (\emptyset, DC)\}$
- Student 2: {(AB,CD),(A,CD),(AB,C),(A,C)}
- Student 3: {(AB,CD)}
- Student 4: {(AB,CD)}

The equilibrium outcome is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & \emptyset & X \end{pmatrix}$, where $X = B$ only when student 1 does not report $B$ as acceptable. Otherwise, $X = \emptyset$.

**Weakly dominant strategies in market SI**

- Student 1: {(AB,DC),(A,DC)}
- Student 2: {(BA,CD),(B,CD)}
- Student 3: {(BA,CD),(BA,C)}
- Student 4: {(AB,DC),(AB,D)}

The unique outcome at the weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ A & B & C & D \end{pmatrix}$, which is the school-optimal stable outcome.

**Weakly dominant strategies in market BOTH**

- Student 1: $\{(BA, DC), (BA, D), (B, D), (\emptyset, D), (AB, DC), (A, DC), (\emptyset, DC), (A, D), (B, DC)\}$
- Student 2: {(BA,DC),(B,DC)}
- Student 3: {(BA,DC),(B,DC)}
- Student 4: {(BA,CD)}

The outcome under weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & B & C & X \end{pmatrix}$, where $X = A$ if (and only if) both students 1 and 3 do not list school $A$ as acceptable to the authority $\{A, B\}$. Otherwise, $X = \emptyset$.

## B.3 DecDA2

**Weakly dominant strategies in market RI**

- Student 1: $\{(AB, DC), (A, DC), (\emptyset, DC), (BA, DC), (B, DC)\}$
- Student 2: {(AB,CD),(A,CD),(AB,C),(A,C)}
- Student 3: {(AB,CD)}
- Student 4: {(AB,CD)}

The unique outcome of the weakly dominant strategies is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & A & B \end{pmatrix}$, which is the student-optimal stable outcome.

**Market SI**

No student has a weakly dominant strategy. There are nine undominated Nash equilibria, which we list below. Note that once offered a choice between schools all students choose a better school according the true preferences in each equilibrium strategy profile.

- All students drop their third choice:

$$\{(B, DC), (A, CD), (BA, D), (AB, C)\}$$

- All students except student 1 drop their third choices:

$$\{(AB, DC), (A, CD), (BA, D), (AB, C)\}$$

- All students except student 2 drop their third choices:

$$\{(B, DC), (BA, CD), (BA, D), (AB, C)\}$$

- All students except student 3 drop their third choices:

$$\{(B, DC), (A, CD), (BA, CD), (AB, C)\}$$

- All students except student 4 drop their third choices:

$$\{(B, DC), (A, CD), (BA, D), (AB, DC)\}$$

- Students 1 and 2 drop their third choices:

$$\{(B, DC), (A, CD), (BA, CD), (AB, DC)\}$$

- Students 1 and 3 drop their third choices:

$$\{(B, DC), (BA, CD), (BA, D), (AB, DC)\}$$

- Students 2 and 4 drop their third choices:

$$\{(AB, DC), (A, CD), (BA, CD), (AB, C)\}$$

- Students 3 and 4 drop their third choices:

$$\{(AB, DC), (BA, CD), (BA, D), (AB, C)\}$$

At the unique equilibrium outcome, each student gets his top choice: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & B & A \end{pmatrix}$.

**Market BOTH**

Weakly dominant strategies of students are as follows:

- Student 1: $\{(BA, DC), (BA, D), (B, D), (\emptyset, D), (AB, DC), (A, DC), (\emptyset, DC), (A, D), (B, DC)\}$
- Student 2: no weakly dominant strategy
- Student 3: no weakly dominant strategy
- Student 4: no weakly dominant strategy

Undominated Nash Equilibria profiles are as follows:

- Student 1: Any strategy submitted to authority $\{C, D\}$ that ranks D at the top and submitted to authority $\{A, B\}$ that either ranks B as unacceptable or ranks A above B.
- Student 4: $(BA, DC)$
- Students 2 and 3 submit the following strategies:
  — $\{(DC, A), (BA, DC)\}$
  — $\{(DC, BA), (BA, D)\}$
  — $\{(DC, A), (B, DC)\}$
  — $\{(DC, B), (BA, D)\}$

or

- Student 1: $\{(BA, DC), (BA, D), (B, D), (\emptyset, D), (\emptyset, DC), (B, DC)\}$

- Student 4: $\{(BA, DC)\}$

- Students 2 and 3 submit following strategies:
— $\{(DC, A), (BA, D)\}$
— $\{(DC, BA), (BA, DC)\}$
— $\{(DC, B), (B, DC)\}$

In the undominated Nash Equilibrium outcome

- Student 1 always receives $D$,

- Student 2 receives $B$ ($C$) if and only if student 3 receives $C$ ($B$),

- Student 4 receives $\emptyset$ if and only if student 3 receives $C$ and ranks $A$ as acceptable.

The Nash equilibria are as follows: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & C & B & A \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & B & C & \emptyset \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 & 3 & 4 \\ D & B & C & A \end{pmatrix}$.

**Remark 3**. One can easily verify that the set of undominated Nash equilibrium profiles under IDA is the same as that of DecDA2 in all of the markets we consider above. However, the (undominated) Nash equilibrium outcomes may be different. Under IDA, the Nash equilibria that include a wasted seat in market BOTH no longer prevails. Besides this, the rest is the same across IDA and DecDA2.

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