Further Inspection of Monty Hall Paradox

Abstract

This paper describes four experiments designed to test different explanations to Monty Hall paradox. Monty Hall paradox is a problem based on TV show, which demonsrates people's inability to correctly apply Bayes reasoning. The experiments that I propose examine the contribution of several factors to the observed behavior: experiment 1 tests the role of status quo; experiment 2 studies if the learning process evolve unconsciously from experience and whether it can be manipulated. Experiment 3 introduces real-life framing of the paradox and examines if it helps subjects to make the right choice. Reaction time is measured to see by how much the reframing simplifies the problem (if at all). Finally, experiment 4 reframes Monty Hall paradox in terms of losses to force subjects to put more cognitive effort. Reaction time is measured to estimate the scope of the effect and to see if more cognitive effort result in improved choices.

1 Introduction

From its very first emergence at Monty Hall's *Let's Make a Deal* television show, the so-called Monty Hall paradox has been subject to fierce debate, which escalated even more when Marilyn vos Savant (1990) published the solution in *Parade* magazine. As Tirney (1991) reports, vos Savant received approximately 10000 critical letters, about 1000 of which were signed by PhD's, including PhD's in mathematics. The original text of the problem is:

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (vos Savant, 1990)

The correct answer, which is that switching leads to higher probability of opening the room with a car, seems completely counterintuitive for prevailing majority of people. Moreover, people do not become convinced even after an accurate explanation. And while this feature makes Monty Hall paradox a good topic for public discussion, its two other features make it a prolific source for scientific research. The first is at least seeming complete irrationality of the subjects' behavior. While many decision making anomalies can be easily explained by introducing non-standard preferences, for instance, loss aversion or prosocial preferences, in this case it is difficult, if not impossible, to come up with realistic preferences driving subjects to choose the door with probability of winning outcome equal to 33%, while the door with 67% chance of winning is available. Another notable feature of the paradox is its somewhat unexpected applicability to market realities. Kluger & Wyatt (2004) show that market setup of Monty Hall paradox may be used to examine whether individual judgement errors regarding asset prices are translated to market equilibrium parameters. Though they contend that this paradox itself has little to do with real-life circumstances, they argue that this is a useful tool to model biased judgements. While Monty Hall paradox is formulated fairly simple, its predictive power and generalizability are strong.

The purpose of the paper is to extend the existing studies, proposing experiments which may differentiate between alternative explanations: experiment 1 examines the "no flip-flopping rule of thumb" introduced by Friedman (1998); experiment 2 sheds light to dual-process explanation of the learning curve (Franco-Watkins, 2003). Other treatments are designed to test whether different framings of the problem may help to diminish the anomaly. Experiment 3 examines whether real-life context simplifies the decision-making process, and experiment 4 shows if reframing the problem in terms of losses, which presumably leads to higher cognitive effort involved, results in higher likelihood of correct answers. The rest of the paper is organized as follows. The next section presents literature review. Then, Section 3 introduces four experimental treatments and discusses the predictions. Section 4 concludes.

2 Literature Review

Monty Hall paradox has received much attention in the literature both in psychology and in experimental economics. While psychological studies are tend to focus more on cognitive mechanisms responsible for the emerging of the paradox (see, for example, Krauss & Wang, 2003; Burns & Wieth, 2004), the experimental economics approach attempts to dismantle the paradox by creating an appropriate learning environment. Friedman (1998) found that when subjects were asked to track how much money they earned and how much money they would have earned if they had sticked to either always stay or always switch strategies, they were more likely to increase the percentage of *switch* in the later periods. The same result was derived when the subjects were told the percentages of other switchers or stickers who won the prize. As one of the potential explanations, Friedman (1998) offers "no flip-flopping" principle. If subjects are used to think that in everyday life it is better not to change their decisions very often, they may continue to do so in a laboratory. However, he does not provide evidence suggesting which of the possible explanations are correct.

Palacious-Huerta (2003) studied the role of social interactions in the learning process of Monty Hall paradox and found that communication could facilitate the learning process, moreover, it helped "less able" subjects more than "more able" subjects. Other authors (Page, 1998, Franco-Watkins et al., 2003) try to implement the idea that with the increased number of doors the problem becomes more straightforward and subjects may be more willing to switch. The results are in line with the hypothesis: Page (1998) confirmed that when choosing among 100 doors subjects switched much more often than when choosing among 10 or 3 doors. However, lack of repeated trials, very small sample sizes (16-17 subjects per treatment) and potential biasedness of the sample (all subjects were MBA students) may present a threat to both internal and external validities. Franco-Watkins et al. (2003) manipulate the numbers of doors and prizes, replicating to some extent the results of Page (1998) and showing that while subjects learn to choose *switch* over time, they fail to correctly estimate probabilities even after a series of trials. They present it as evidence in support of two-systems approach: system 1 learns what to do, while system 2 remains unaware of the causes.

This paper aims to, following the framework set by Friedman (1998) and Franco-Watkins et al. (2003), examine further Monty Hall paradox and the learning process. Experiments 1 and 2 try to find evidence supporting or contradicting the explanations proposed in the previous studies, and experiments 3 and 4 introduce new setups of the paradox, which may help to shed light on its nature. Another contribution of this paper is that in all experiments reaction time of the subjects is measured, which has not been done before. Following Rubinstein (2016), reaction time serves here as a proxy of cognitive effort involved into the decision making process. Differentiation between levels of cognitive effort is particularly important for experiments 3 and 4.

3 Experimental Treatments

All the treatments listed below are variations of the baseline three-doors Monty Hall problem. The baseline treatment serves as a control for the remaining treatments. The experiments are between-subjects; each participant receives $5 show-up fee. Each treatment consists of 20 identical trials. Before a series of trials, the experimenter reads all the instructions aloud and demonstrates an example. For each trial and each subject, response time is measured.

In the instructions for a trial in the baseline treatment I closely follow Franco-Watkins et al. (2003). Comments regarding subject's actions appear in parentheses:

> You are a contestant on a game show. A game-show host has randomly placed a new car behind one of three identical doors. There is a goat behind each of the other doors. Now you get to select a door.
>
> (The subject selects a door at her screen)
>
> After you have selected your door, the host, who knows where the car is, opens up one of the other two doors to show that the car is not behind that door. He will always show you a door that has a goat behind it, and he will never open up your door.
>
> (One of the two unchosen doors with a goat behind it opens at the subject's screen)
>
> Now the door which you have chosen will be opened. You also have an option to switch and open the door that you did not select and the host did not open. In case if there is a car behind the opened door, you will receive $1. Otherwise, you will receive 0.
>
> (The subject chooses to stay or switch. The chosen door opens; the subject is told how much she earned in this round and in all previous rounds).

The prediction here is that the results of the previous studies would be replicated: learning over time would be observed, starting from about 10% of switches in the first periods and ending with about 30% of switches in the last periods.

3.1 Treatment 1

This experiment aims to differentiate between "no flip-flopping" hypothesis presented in Friedman (1998) and "feeling lucky" hypothesis presented in Page (1998). The only difference between this treatment and the baseline treatment is the default option on the last step. In this treatment, the door which the subject did not select on the first stage and the host did not open on the second stage is opened by default. The subject may switch and open the door which she selected at the first stage. That is, the probabilistic representation of the problem is exactly the same, but now the correct solution is default:

> You are a contestant on a game show. A game-show host has randomly placed a new car behind one of three identical doors. There is a goat behind each of the other doors. Now you get to select a door.
>
> (The subject selects a door at her screen)

After you have selected your door, the host, who knows where the car is, opens up one of the other two doors to show that the car is not behind that door. He will always show you a door that has a goat behind it, and he will never open up your door.

(One of the two unchosen doors with a goat behind it opens at the subject's screen)

Now the door which you you did not select and the host did not open will be opened. You also have an option to switch and open the door that you selected in the beginning. In case if there is a car behind the opened door, you will receive $1. Otherwise, you will receive 0.

(The subject chooses to stay or switch. The chosen door opens; the subject is told how much she earned in this round and in all previous rounds).

Note that if "feeling lucky" hypothesis is correct, and the subjects feel something special about the door they initially choose, then, the prediction is that the frequences of *switch* and *stay* should be inverted compared to the baseline frequences. That is, subjects will still stick to the door they choose in the first stage, even if now the options are labeled differently.

However, if "no flip-flopping" is the right explanation (the subjects do not like to change the default option) then the frequences of *switch* and *stay* should be similar to frequences of the baseline treatment. I would expect that the latter would happen, since most subjects report that they estimate the probabilities of the remaining two doors to cover the prize equal, in other words, they do not believe that there is something special about the door they have selected (Page, 1998).

3.2 Treatment 2

This experiment tests the hypothesis presented by Franco-Watkins et al. (2003). They argue that dual-process decision making might take place here: subjects seem to learn to choose the correct option, guided by "gut feeling", but they do not acquire understanding of probabilities over time. That is, they unconsciously learn from experience that switching strategy leads to better results but do not develop a mental model for it. An alternative explanation here might be that subjects nevertheless eventually build the correct model of the problem, they simply cannot formalize and articulate it via probabilistic language. The two alternatives can be distinguished by manipulating the learning process.

In the three subtreatments, assignment of the prize to doors will not be random anymore. The assignment will happen after the initial selection of a door by the subject. Aside from this, everything else will be just like in the baseline treatment. In the first subtreatment the door that subject chose will be assigned with the prize with probability equal to $\frac{1}{3}$. Note that this subtreatment is equivalent to the baseline treatment. This is added to ensure that learning process manipulation does not create any distortions besides planned. The prediction here is that the results will be perfectly in line with the baseline treatment.

In the second treatment the prize will be assigned to the chosen door with probability equal to $\frac{1}{2}$, which corresponds to the estimation of probability reported by most subjects (Page, 1998). Since the observed in the baseline treatment behavior of subjects is presumably their reaction to such a belief, the prediction of the "dual process" approach here is that there will be less learning effects compared to the first subtreatment. If the alternative hypothesis is correct and the understanding of the problem enhances over time regardless of experience, learning effects will persist.

In the third treatment the prize will be assigned to the chosen door with probability equal to $\frac{2}{3}$, which inverts the correct probability. The prediction of the "dual-process" approach here is that subjects will not learn to switch or even will learn to stay, while the prediction of the alternative hypothesis is that learning curve will be similar to ones derived in the first two subtreatments.

One might argue that this treatment involves deception of subjects, which, as we know, is not encouraged in experimantal economics (Hertwigg & Ortmann, 2001). I would object that in this case the degree of deception is not even close to that high as in experimental psychology studies. Indeed, the manipulated patterns of winning and losing outcomes may well be a result of the honest baseline treatment. For example, one might easily calculate that the chances to guess the correct door in 10 or more trials out of 20 is 9.1%, which is substantial. If one still insist that even such an innocent deception of the subjects is unacceptable, I propose an alternative: the experimenter should conduct the baseline treatment on thousands of subjects. For some of them the frequences of correct guesses in the first stage will inevitably be close to 50% and 67%. Then the experimenter should create dummies corresponding to the frequences of correct guesses (33% or less, about 50%, 67% or more), regress switching on them and the number of round, then look at the interaction of the dummies with the number of round. The "dual-process" approach predicts that coefficients will decrease from 33% dummy to 67%, while the alternative hypothesis predicts that the coefficients will be equal.

3.3 Treatment 3

This treatment attempts to reframe the problem in a real-life context. The motivation for it is that people usually perform better and make better decisions in familiar to them situations, while the setup of Monty Hall paradox is arguably pretty artificial. In this treatment response time becomes crucial. If the real-life reframing helps indeed, we should observe two effects: higher percentage of *switch* and lower response time. The instructions for treatment 3 are as follows:

You consider buying a stock from one of the three companies for a very low price. You know for sure that due to the bubble in the market two of them will get bankrupt next year and will not bring you profit, while the remaining one will. However, you do not know which of the three companies will survive. Now you get to select a company.

(The subject selects a stock at her screen)

After you have selected your stock, you encounter a financial expert, who knows what happens inside the companies and in the market and knows which company will survive. He reveals one of the remaining two companies which is going to get bankrupt for sure. The financial expert always names a company which will get bankrupt, and he is not allowed to tell you anything about the company you chose.

(One of the two unchosen stocks is labeled bankrupt at the subject's screen)

Now you may stay with your stock or you have an opportunity to exchange your stock to the stock which you did not choose and the financial expert did not reveal. In case if the chosen company survives, you will receive $1. Otherwise, you will receive 0.

(The subject chooses to stay or switch. The future of the chosen company is revealed; the subject is told how much she earned in this round and in all previous rounds).

3.4 Treatment 4

The last treatment, similarly to the previous one, tries to influence cognitive effort. But while treatment 3 aimed to decrease the threshold level of cognitive effort needed to correctly solve the problem, treatment 4 tries to force subjects to think harder. Palacious-Huerta (2003) used intensified incentives in order to achieve higher cognitive effort, and Franco-Watkins et al. (2003) used narrow-broad framing (participants in one group made decisions each round, while participants in the other group made a choice for a block of 5 trials). The results seem ambiguous. This treatment tries to exploit loss aversion to increase cognitive effort.

Since people do not like losses, they are expected to consider more thoroughly the possible choices. Thus, the prediction here is higher response time compared to the baseline treatment. The second question is whether more cognitive effort involved results in better decisions. If indeed people are able to come to the right answer after more careful assessment of probabilities, then we should see the higher percentage of switches compared to the baseline treatment. But if higher effort does not lead to correct conclusions, we will see that while the response time is slower, the frequences of choices are the same as in the baseline treatment. However, it is also possible that even if in the first periods the percentage of switches will be the same, the loss aversion phenomenon will facilitate the learning process. I would expect the latter path to actually happen. The instructions for the treatment follow:

You get $1 in addition to your account in the beginning of this round.

(A dollar is added to the subject's account displayed on the screen)

Imagine you are a contestant on a game show. A game-show host has randomly placed a new car behind one of three identical doors. There is a goat behind each of the other doors. Now you get to select a door.

(The subject selects a door at her screen)

After you have selected your door, the host, who knows where the car is, opens up one of the other two doors to show that the car is not behind that door. He will always show you a door that has a goat behind it, and he will never open up your door.

(One of the two unchosen doors with a goat behind it opens at the subject's screen)

Now the door which you have chosen will be opened. You also have an option to switch and open the door that you did not select and the host did not open. In case if there is a car behind the opened door, you will save to your account the dollar that was given to you in the beginning of the round. Otherwise, you will lose the dollar.

(The subject chooses to stay or switch. The chosen door opens; the subject is told how much she earned in this round and in all previous rounds).

4 Conclusion

The experiments presented in this paper may help to learn more about the nature of Monty Hall paradox. However, the two major questions remain. Firstly, what generally drives the problems of this kind? The examined paradox is just one example in the range of problems demonstrating people's inability to correctly apply Bayes reasoning. Further research might study all these problems together to search for common expaining factors or common learning curve. Secondly, what implications do these problems (and Monty Hall paradox particularly) have on the real life? Do people actually suffer from lack of Bayes logic? Field experiments will probably be needed to address these questions.

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