

Game theory experiments

November 6, 2020

What is a game

- ▶ A **normal form game** is $(I, (A^i)_{i=1,\dots,n}, (u^i)_{i=1,\dots,n})$
- ▶ $I = \{1, \dots, n\}$ is the set of players
- ▶ $\forall i$ A_i is an **action set** of player i , $A = \times_{i=1}^n A^i$
- ▶ $u_i : A \rightarrow \mathbb{R} \forall i$ if a **utility function** of player i

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 - ▶ If the average is 23, you win by picking 15
- ▶ No decimals or fractions allowed

Another simple game

	A	B
A	3, 3	0, 4
B	4, 0	2, 2

Another simple game

	C	D
C	3, 3	0, 4
D	4, 0	2, 2

Prisoner's dilemma

	C	D
C	-7, -7	-10, -6
D	-6, -10	-8, -8

Dominance

- ▶ An action a^i **strictly dominates** an action b^i if and only if

$$u^i(a^i, a^{-i}) > u^i(b^i, a^{-i}) \quad \text{for all } a^{-i} \in A^{-i}$$

Probabilistic strategies

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- ▶ We say that b^i is **strictly dominated** if there exists some mixed strategy that dominates it

Theorem

An action is strictly dominated if and only if it is **never a best response** (never rational)

Iterated Elimination of Strictly Dominated Actions

Another example

		Player 2			
		b_1	b_2	b_3	b_4
Player 1	a_1	0, 7	2, 5	7, 0	0, 1
	a_2	5, 2	3, 3	5, 2	0, 1
	a_3	7, 0	2, 5	0, 7	0, 1
	a_4	0, 0	0, -2	0, 0	10, -1

Another example

- ▶ **Guessing game:**

- ▶ Everything above $2/3 * (100)$ eliminated since it is never a best response
- ▶ Everything above $(2/3)^2 * (100)$ eliminated in the remaining game
- ▶ etc.

Level-k model

- ▶ Level-0 responds randomly
- ▶ Level-1 best responds to Level-0
- ▶ Level-2 best responds to Level-1
- ▶ etc