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Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure

By JOHN B. VAN HUYCK, RAYMOND C. BATTALIO, AND RICHARD O. BEIL*

Deductive equilibrium methods—such as Rational Expectations or Bayesian Nash Equilibrium—are powerful tools for analyzing economies that exhibit strategic interdependence. Typically, deductive equilibrium analysis does not explain the process by which decision makers acquire equilibrium beliefs. The presumption is that actual economies have achieved a steady state. In economies with stable and unique equilibrium points, the influence of inconsistent beliefs and, hence, actions would disappear over time, see Robert Lucas (1987). The power of the equilibrium method derives from its ability to abstract from the complicated dynamic process that induces equilibrium and to abstract from the historical accident that initiated the process.

Unfortunately, deductive equilibrium analysis often fails to determine a unique equilibrium solution in many economies and, hence, often fails to prescribe or predict rational behavior. In economies with multiple equilibria, the rational decision maker formulating beliefs using deductive equilibrium concepts is uncertain which equilibrium strategy other decision makers will use and, when the equilibria are not interchangeable, this uncertainty will influence the rational decision-maker's behavior. Strategic uncertainty arises even in situations where objectives, feasible strategies, institutions, and equilibrium conventions are completely specified and are common knowledge. While multiple equilibria are common in theoretical analysis, consideration of specific economies suggests that many equilibrium points are implausible and unlikely to be observed in actual economies.

One response to multiple equilibria is to argue that some Nash equilibrium points are not self-enforcing and, hence, are implausible, because they fail to satisfy one or more of the following refinements: elimination of individually unreasonable actions, sequential rationality, and stability against perturbations of the game—see Elon Kohlburg and Jean-Francois Mertens (1986) for examples and references. Equilibrium refinements determine when an outcome that is *already expected* would be implemented by rational decision makers.

In general, many outcomes will satisfy the conditions of a given equilibrium refinement. The equilibrium selection literature attempts to determine which, if any, self-enforcing equilibrium point will be expected. A satisfactory theory of interdependent decisions must not only identify the outcomes that are self-enforcing when expected but also must identify the expected outcomes. Consequently, a theory of equilibrium selection would be a useful complement to the theory of equilibrium points.

The experimental method provides a tractable and constructive approach to the equilibrium selection problem. This paper studies a class of tacit pure coordination games with multiple equilibria, which are

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strictly Pareto ranked, and it reports experiments that provide evidence on how human subjects make decisions under conditions of strategic uncertainty.

I. A Pure Coordination Game

To focus the analysis consider the following tacit coordination game, which is a strategic form representation of John Bryant's (1983) Keynesian coordination game. The baseline game is defined as follows: Let $e_1, \dots, e_n$ denote the actions taken by n players. The period game A is defined by the following payoff function and strategy space for each of n players:

$$(1) \quad \pi(e_i, \underline{e}_i) = a[\min(e_i, \underline{e}_i)] - be_i, \quad a > b > 0,$$

where $\underline{e}_i$ equals $\min(e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n)$. Actions are restricted to the set of integers from 1 to $\bar{e}$. The players have complete information about the payoff function and strategy space and know that the payoff function and strategy space are common knowledge.[1]

If the players could explicitly coordinate their actions, the—real or imagined—planner's decision problem would be trivial. Given $a - b$ greater than 0, each player should choose the maximum feasible action, $\bar{e}$. Moreover, a negotiated "pregame" agreement to choose $\bar{e}$ would be self-enforcing. Unlike games with incentive problems, here the first best outcome is an equilibrium point. However, when the players cannot engage in "pregame" negotiation they face a nontrivial coordination problem.

Suppose that the players attempt to use the Nash equilibrium concept to inform their strategic behavior in the tacit coordination game A. A player's best response to $\underline{e}_i$ is to choose e_i equal to $\underline{e}_i$. By symmetry it follows that any n-tuple $(e, \dots, e)$ with $e \in \{1, 2, \dots, \bar{e}\}$ satisfies the mutual best response property of a Nash equilibrium point. All feasible actions are potential Nash equilibrium outcomes. The Nash concept neither prescribes nor predicts the outcome of this tacit coordination game.

(Standard equilibrium refinements do not reduce the set of equilibria. For example, the equilibria are strict—each player has a unique best response—and, hence, trembling-hand perfect.)

II. Coordination Problems and Equilibrium Selection Principles

The analysis in Section I follows convention and abstracts from the equilibrium selection problem. However, a rational player using deductive equilibrium concepts confronts two nontrivial coordination problems in period game A. First, players may fail to correctly forecast the minimum, $\underline{e}_i$, and, hence, regret their individual choice, that is, $e_i \neq \underline{e}_i$. This type of coordination failure results in disequilibrium: outcomes that do not satisfy the mutual best-response property of an equilibrium.

An equilibrium selection principle identifies a subset of equilibrium points according to some distinctive characteristic. An interesting conjecture is that decision makers use some selection principle to identify a specific equilibrium point in situations involving multiple equilibria. This selection principle would solve the problem of coordinating on a specific equilibrium point. Hence, the outcome will satisfy the mutual best-response property of an equilibrium.

A second coordination problem arises when the equilibria can be Pareto ranked. In such situations, all players may give a best response, but, nevertheless, implement a

[1] Apparently, this game is similar to Rousseau's "stag hunt" parable, which he used to motivate his analysis of the social contract, see Crawford (1989, p. 4). In the stag hunt game, each hunter in a group must allocate effort between hunting a stag with the group and hunting rabbits by himself. Let e_i denote effort expended on the stag hunt. Since stag hunting in that era required the coordinated effort of *all* the hunters, the probability of successfully hunting a stag depends on the smallest e_i. The parameter a in equation (1) reflects the benefits of participating in the stag hunt: eating well should the hunt succeed. Hunting rabbits does not require coordination with the other hunters. The parameter b in equation (1) reflects the opportunity cost of effort allocated to the stag hunt that could have been allocated to rabbit hunting: a meal—however, meager.

Pareto dominated equilibrium, that is, $\min(e_1,\dots,e_n) \neq \bar{e}$. While not regretting their individual choice, they regret the equilibrium implemented by these individual choices. Consequently, the outcome results in coordination failure. What equilibrium selection principles could a player use to resolve these two coordination problems?[2]

Deductive selection principles select equilibrium points based on the description of the game. Deductive selection principles preserve the equilibrium method's desirable property of independence from historical accidents and from complicated dynamic processes. Inductive selection principles select equilibrium points based on the history of some pregame.[3] Hence, inductive selection principles are not independent of accident and process.

When multiple equilibrium points can be Pareto ranked, it is possible to use concepts of efficiency to select a subset of self-enforcing equilibrium points: examples include R. Duncan Luce and Howard Raiffa's (1956, p. 106) concept of joint-admissibility, Tamer Basar and Geert Olsder's (1982, p. 72) concept of admissibility, and John Harsanyi and Reinhard Selten's (1988, p. 81) concept of payoff-dominance. An equilibrium point is said to be payoff-dominant if it is not strictly Pareto dominated by any other equilibrium point. When unique, considerations of efficiency may induce players to focus on and, hence, select the payoff-dominant equilibrium point, see Thomas Schelling (1980, p. 291).

In period game A, the equilibrium points are strictly Pareto ranked. Each player prefers a larger minimum. The only equilibrium point not Pareto dominated by any other equilibrium point is the n-tuple $(\bar{e},\dots,\bar{e})$: the payoff-dominant equilibrium point. Consequently, payoff-dominance selects the n-tuple $(\bar{e},\dots,\bar{e})$ in game A.

Selecting the unique payoff-dominant equilibrium point not only allows players to coordinate on an equilibrium point but also ensures that they will not coordinate on an inefficient one. Payoff dominance solves both the individual and the collective coordination problems of disequilibrium and coordination failure and, as Harsanyi and Selten suggest, *should* take precedence over alternative selection principles.

The tacit coordination game A provides a severe test of payoff dominance, because the minimum rule exacerbates the influence of uncertainty about the strategies of the other $n-1$ players. Define the cumulative distribution function for a player's action as $F(e_j)$. In the payoff-dominant equilibrium, $F(\bar{e})$ equals 1 and $F(e_j)$ equals 0 for e_j less than $\bar{e}$. A well-known theorem is that if $e_1,\dots,e_n$ are independent and identically distributed with common cumulative distribution function $F(e_j)$, then the cumulative distribution function for the minimum, $F_{\min}(e)$, equals $1-[1-F(e_j)]^n$; see A. M. Mood, F. A. Graybill, and D. C. Boes (1974). In the payoff-dominant equilibrium, $F_{\min}(\bar{e})$ equals 1 and $F_{\min}(e)$ equals 0 for e less than $\bar{e}$. But suppose that a player is uncertain that the $n-1$ players will select the payoff-dominant action, $\bar{e}$. Specifically, let $F(1)$ be small but greater than 0, then as n goes to infinity $F_{\min}(1)$ goes to 1. Consequently, when the number of players is large it only takes a remote possibility that an individual player will not select the payoff-dominant action $\bar{e}$ to motivate defection from the payoff-dominant equilibrium.

Several deductive selection principles based on the "riskiness" of an equilibrium point have been identified and formalized. A maximin action, which is an action (pure strategy) with the largest payoff in the worst possible outcome, is secure, see John Von Neumann and Oskar Morgenstern (1944, 1972). Given existence, security selects the equilibrium point supported by player's maximin actions. Security may select very inefficient equilibrium points in nonzero sum games.

In period game A, a player can ensure a payoff of $a-b$ by choosing e_i equal to 1, which is the largest payoff in the worst possi-

[2] The "disequilibrium Keynesians" emphasize the first coordination problem. The "equilibrium Keynesians" emphasize the second coordination problem, see Cooper and John (1988) for examples and references.

[3] We use the term induction in the logical, rather than mathematical, sense of reasoning from observed facts—history—to a conclusion.

TABLE 1—EXPERIMENTAL DESIGN

Experiment No.	Date	Size	*A* Payoff *A* Fullsize	*B* Payoff *B* Fullsize	*A′* Payoff *A* Fullsize	*C* Payoff *A* Size Two[a]
1	June	16	$1^p,2,\ldots,10$	–	–	–
2	June	16	$1^p,2,\ldots,10^p$	$11,\ldots,15$	$16^p,\ldots,20$	–
3	June	14	$1^p,2,\ldots,10^p$	$11,\ldots,15$	$16^p,\ldots,20$	–
4	Sept	15	$1^p,2^p,\ldots,10^p$	$11^p,\ldots,15$	$16,\ldots,20$	$21,\ldots,27$
5	Sept	16	$1^p,2^p,\ldots,10^p$	$11^p,\ldots,15$	$16,\ldots,20$	$21,\ldots,27$
6	Sept	16	$1^p,2^p,\ldots,10^p$	$11^p,\ldots,15$	$16,\ldots,20$	$21,\ldots,25$
7	Sept	14	$1^p,2^p,\ldots,10^p$	$11^p,\ldots,15$	$16,\ldots,22$	$23,\ldots,25$

^{p} ~ Denotes a period in which subjects made predictions.
[a] ~ In experiment 4 and 5 pairings were fixed, while in experiments 6 and 7 pairings were random.

ble outcome. Consequently, in this tacit coordination game, security selects the n-tuple $(1,\ldots,1)$. Since payoff-dominance and security select different equilibrium points in this tacit coordination game (equilibrium points with the highest and lowest payoffs, respectively), an important and tractable empirical question is which, if any, deductive selection principle organizes the experimental data.

It is often possible to apply more than one deductive selection principle to a game. Hence, subjects may choose disequilibrium outcomes unless they behave as if there is a hierarchy of selection principles. When deductive selection principles fail to coordinate beliefs and actions, inductive selection principles based on repeated interaction may allow players to *learn to coordinate*.

Consider a finitely repeated game $A(T)$, which involves the n players playing period game A for T periods. The payoff-dominant equilibrium of $A(T)$ is just the repeated implementation of the payoff-dominant equilibrium of period game A, because the first-best outcome for period game A is $(\bar{e},\ldots,\bar{e})$. Similarly, the secure equilibrium of $A(T)$ is just the repeated implementation of the secure equilibrium of period game A.[4]

[4]Crawford (1989) emphasizes that the secure equilibrium is also the only equilibrium that is evolutionary stable. In repeated play, players using adaptive behavior may be led to implement the secure equilibrium. Hence, while the experiments reported in this paper discriminate sharply between strategic stability and evolutionary stability, they do not discriminate between learning to use security and certain kinds of adaptive behavior.

Notice that the principles of efficiency and security can be defined independently of equilibrium. The experiments in this paper, which are designed to study the conflict between efficiency and security, are *not* designed to study how repeated play of period game A influences the set of equilibrium points for $A(T)$.

Having t periods of experience in $A(T)$ provides a player with observed facts, in addition to the description of the game, that can be used to reason about the equilibrium selection problem in the continuation game $A(T-t)$. This experience may influence the outcome of the continuation game $A(T-t)$ by focusing expectations on a specific equilibrium point. For example, one adaptive hypothesis is that players will give a best response to the minimum observed in the previous period. This adaptive behavior would immediately converge to an equilibrium in $A(T-1)$. The selected equilibrium involves all players choosing the period 1 minimum for the $T-1$ periods of the continuation game $A(T-1)$.

III. Experimental Design

Table 1 outlines the design of the seven experiments reported in this paper. The instructions were read aloud to ensure that the description of the game was common information, if not, common knowledge.[5] No pre-

[5]The original working paper, Van Huyck, Battalio, and Beil (1987), which includes the actual instructions,

PAYOFF TABLE A

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
Choice	6	–	1.20	1.00	0.80	0.60	0.40	0.20
of	5	–	–	1.10	0.90	0.70	0.50	0.30
X	4	–	–	–	1.00	0.80	0.60	0.40
	3	–	–	–	–	0.90	0.70	0.50
	2	–	–	–	–	–	0.80	0.60
	1	–	–	–	–	–	–	0.70

PAYOFF TABLE B

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.20	1.10	1.00	0.90	0.80	0.70
Choice	6	–	1.20	1.10	1.00	0.90	0.80	0.70
of	5	–	–	1.10	1.00	0.90	0.80	0.70
X	4	–	–	–	1.00	0.90	0.80	0.70
X	3	–	–	–	–	0.90	0.80	0.70
	2	–	–	–	–	–	0.80	0.70
	1	–	–	–	–	–	–	0.70

play negotiation was allowed. After each repetition of the period game, the minimum action was publicly announced and the subjects calculated their earnings for that period. The only common historical data available to the subjects was the minimum.

During the course of an experiment some design parameters were altered resulting in a sequence of treatments labeled A, B, A', and C. Instructions for continuation treatments were given to the subjects after earlier treatments had been completed. The feasible actions in all treatments of all experiments were the integers 1 through 7: hence, $\bar{e}$ equaled 7.

In treatment A and A', the following values were assigned to the parameters in equation (1): parameter a was set equal to \$0.20, parameter b was set equal to \$0.10, and a constant of \$0.60 was added to ensure that all payoffs were strictly positive. Consequently, the payoff-dominant equilibrium, $(7,\dots,7)$, paid \$1.30 while the secure equilibrium, $(1,\dots,1)$, paid \$0.70 per subject per period.[6] Subjects were given this information in the form of a payoff table, see payoff Table A. In treatment A, the period game A was repeated ten times. The number of players, n, varied between 14 and 16 subjects. (In treatment C, the number of players, n, was reduced to two.) Treatment A' designates the resumption of these conditions after treatment B.

In treatment B, parameter b in equation (1) was set equal to zero, see payoff Table B. This gives the subjects a dominating strategy (play 7 regardless of the minimum), which eliminates the coordination problem. The number of players, n, remained the same as in treatment A.

payoff tables, questionnaire, extra instructions and record sheet used in the experiments and a more extensive analysis of the experimental results, is available from the authors upon request.

[6]For the remainder of this paper, an equilibrium denotes a mutual best-response *outcome* in the period game.

Occasionally, subjects were asked to predict the actions of all the subjects in the treatment.[7] For each prediction in the September experiments, a subject was paid \$0.70 less 0.02 times the sum of the absolute value of the difference between the actual and predicted actions. (The rule used in the June experiments was less sensitive to prediction errors). At the end of the experiment, the subjects were told the actual distribution of actions and were paid.

The subjects were undergraduate students attending Texas A&M University and were recruited form sophomore and junior economics courses. A total of 107 students participated in the seven experiments. After reading the instructions, but before the experiment began, the students filled out a questionnaire to determine that they understood how to read the payoff table for treatment A, that is, map actions into money payoffs. The instructions would have been re-read if needed, but all 107 students responded correctly.

IV. Experimental Results

Table 2 reports the experimental results for treatment A. The data in period one are particularly interesting because the subjects can only use deductive selection principles to inform their behavior.

In period one, the payoff-dominant action, 7, was chosen by 31 percent of the subjects (33 of 107) and the secure action, 1, was chosen by 2 percent of the subjects (2 of 107). Neither deductive selection principle succeeds in organizing much of the data, although payoff-dominance is more successful than security. The popularity of actions 4 and 5—chosen by 18 and 34 subjects, respectively—is consistent with many subjects having nearly diffuse prior beliefs about the outcome of the period game.

The initial play of all seven experiments exhibit both individual and collective coordination failure. The minimum action for period one was never greater than 4. Hence, the largest payoff in period one was \$1.00 and some payoffs were \$0.10. (The payoff-dominant equilibrium would have paid everyone \$1.30). All of these outcomes are inefficient. The subjects were unable to use any deductive selection principle to coordinate on an equilibrium point.

Only 10 percent of the subjects predicted an equilibrium outcome in period one. Instead, most subjects (95 of 106) predicted a disequilibrium outcome.[8] Moreover, the subjects' predictions were dispersed: one third of the subjects predicted at least one 1 and one 7—a range of 6—and the average range of the predictions was 4.0. The subjects' dispersed predictions suggest that they expected other subjects to respond to the payoff table differently than they did. These data are inconsistent with any theory of equilibrium selection that assumes that, because a player will derive his prior probability distribution over other players' pure strategies strictly from the parameters of the game, all players will have the same prior probability distribution. Instead, some subjects made optimistic predictions and some subjects made pessimistic predictions.

While the subjects were unable to coordinate beliefs and actions, in almost all cases their individual predictions and actions were consistent. Of the 107 subjects, 106 subjects predicted that at least one other subject would choose an action equal to or less than their choice. (Only one subject predicted he would determine the minimum.) Most subjects mapped predictions into actions in a reasonable way. Those subjects who made pessimistic predictions about what the other subjects would do chose small values for their action and subjects who made optimistic predictions about what the other subjects would do chose large values for their action.[9]

[7]In two earlier pilot experiments predictions were not made in any period. The substantive results were the same as those reported here.

[8]One subject is excluded due to predicting only 15 choices.

[9]An anomaly is that, of the 95 subjects who predicted a minimum less than 7, 87 subjects chose an action greater than the minimum they predicted. Van Huyck, Battalio, and Beil (1987) provide an expected value model to explain this anomaly.

TABLE 2—EXPERIMENTAL RESULTS FOR TREATMENT *A*

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 1										
No. of 7's	8	1	1	0	0	0	0	0	0	1
No. of 6's	3	2	1	0	0	0	0	0	0	0
No. of 5's	2	3	2	1	0	0	1	0	0	0
No. of 4's	1	6	5	4	1	1	1	0	0	0
No. of 3's	1	2	5	5	4	1	1	1	0	1
No. of 2's	1	2	2	4	8	7	8	6	4	1
No. of 1's	0	0	0	2	3	7	5	9	12	13
Minimum	2	2	2	1	1	1	1	1	1	1
Experiment 2										
No. of 7's	4	0	1	0	0	0	0	0	0	1
No. of 6's	1	0	1	0	0	1	0	0	0	0
No. of 5's	3	3	2	1	0	0	1	1	0	1
No. of 4's	4	6	2	3	3	0	0	0	0	0
No. of 3's	1	4	2	5	0	1	1	0	1	0
No. of 2's	3	2	6	5	5	9	3	4	3	1
No. of 1's	0	1	2	2	8	5	11	11	12	13
Minimum	2	1	1	1	1	1	1	1	1	1
Experiment 3										
No. of 7's	4	4	1	0	1	1	1	0	0	2
No. of 6's	2	0	2	0	0	0	0	0	0	0
No. of 5's	5	6	1	1	1	0	0	0	0	0
No. of 4's	3	3	2	1	2	1	0	0	0	1
No. of 3's	0	0	7	6	0	2	3	0	0	0
No. of 2's	0	1	1	4	5	3	6	3	2	2
No. of 1's	0	0	0	2	5	7	4	11	12	9
Minimum	4	2	2	1	1	1	1	1	1	1
Experiment 4										
No. of 7's	6	0	1	1	0	0	1	0	0	0
No. of 6's	0	6	2	0	0	1	0	0	0	0
No. of 5's	8	5	5	5	0	1	0	0	0	0
No. of 4's	1	1	4	6	7	1	2	1	1	0
No. of 3's	0	2	3	2	4	3	2	2	1	0
No. of 2's	0	1	0	0	2	3	7	4	2	2
No. of 1's	0	0	0	1	2	6	3	8	11	13
Minimum	4	2	3	1	1	1	1	1	1	1

An interesting question is whether the subjects' predictions correspond to the actual distribution of actions more closely than predictions based on payoff-dominance or security. Using the number of actions correctly predicted as a statistic, the data reveal that 95 percent of the subjects predicted the actions of the other $n-1$ subjects more accurately than did payoff-dominance. This statistic is used to measure prediction accuracy because the subjects payoff were a linear transformation of the prediction accuracy score. The difference of the mean prediction accuracy score was always positive and in most cases significantly different from zero at the 1 percent level: a result that is robust to non-parametric statistical procedures. Obviously, security does even worse. Subjects predicted the observed heterogenous response to the description of the game and the resulting coordination failure in period one.

Repeated play of the period game allows the subjects to use inductive selection principles or to learn to use deductive selection principles. Hence, repeated play makes it more likely that subjects will be able to obtain mutual best-response outcomes in the continuation game. Repeating the period game does cause actions to converge to a

TABLE 2—EXPERIMENTAL RESULTS FOR TREATMENT A, Continued

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 5										
No. of 7's	2	2	3	1	1	1	1	0	0	0
No. of 6's	1	3	1	0	0	0	0	0	0	0
No. of 5's	9	3	0	4	1	0	2	0	0	0
No. of 4's	3	4	6	2	1	2	0	2	1	1
No. of 3's	1	2	2	4	6	0	0	0	0	1
No. of 2's	0	2	2	3	4	6	5	2	5	3
No. of 1's	0	0	2	2	3	7	8	12	10	11
Minimum	3	2	1	1	1	1	1	1	1	1
Experiment 6										
No. of 7's	5	3	1	1	1	1	2	2	2	3
No. of 6's	2	0	0	0	1	0	0	0	0	0
No. of 5's	5	1	0	0	0	1	0	0	0	0
No. of 4's	2	3	4	0	0	0	0	0	0	0
No. of 3's	1	5	4	2	2	2	1	0	2	0
No. of 2's	0	2	4	5	3	3	6	4	5	5
No. of 1's	1	2	3	8	9	9	7	10	7	8
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 7										
No. of 7's	4	3	1	1	1	1	1	1	1	1
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	2	3	0	0	0	0	0	0	0	0
No. of 4's	4	0	1	2	1	0	0	0	0	0
No. of 3's	1	3	2	1	1	0	0	0	0	0
No. of 2's	1	3	2	2	4	4	4	4	5	3
No. of 1's	1	2	8	8	7	9	9	9	8	10
Minimum	1	1	1	1	1	1	1	1	1	1

stable outcome, see Table 2. But rather than converging to the payoff-dominant equilibrium or to the initial outcome of the treatment, the most inefficient outcome obtains in all seven experiments.

The change in a subject's action between period one and period two provides insight into the subjects' dynamic behavior. Of the eleven subjects who determined the minimum in period one the average change in action between period one and two was 0.73: seven subjects increased their action, three did not change, and one decreased his action. In every experiment someone who had not determined the minimum in period one determines the minimum in period two. Moreover, in experiments one through five the intersection of the set of subjects who determine the minimum in period one with the set of subjects who determine the minimum in period two is empty. Since a subject's payoff is increasing in e_i when he (she) uniquely determines the minimum, this adaptive behavior can be rationalized.

However, a subject's payoff is decreasing in e_i when he (she) played above the minimum and those subjects that played above the minimum reduced their choice of action. The observed mean reduction is increasing in the difference between a subject's action and the reported minimum, and the mean reduction is smaller than this difference. The observed correlation between the current choice of these optimistic subjects and the minimum reported in period 1 suggests that behavior in the continuation game $A(9)$ is not independent of the history leading up to continuation game $A(9)$. However, only 14 of 107 subjects give a best-response to the period one minimum in period two.

Some subjects play below the minimum of the preceding period. This observed "overshooting" cannot be reconciled with adaptive theories that predict the current action will be a convex combination of last periods action and last periods outcome. Apparently, some subjects learn how "risky" it is to choose an action other than the secure action, 1, under the minimum rule and learn to use security to inform their behavior in the continuation game.

Although it failed to predict the initial outcome, security predicts the stable outcome of period game A. By period ten 72 percent of the subjects (77 out of 107) adopt their secure action, 1, and the minimum for all seven experiments was a 1. The observed coordination failure appears to result from a few subjects concluding it is too "risky" to choose an action other than the secure action and from most subjects focusing on the minimum reported in earlier period games. The minimum rule interacting with this dynamic behavior causes this treatment to converge to the most inefficient outcome.

In treatment B, parameter b of equation (1) was set equal to zero. Because a player's action is no longer penalized, the payoff-dominant action, 7, is a best response to all feasible minimums. Action 7 is a dominating strategy. Hence, treatment B tests equilibrium *refinements* based on the elimination of individually unreasonable actions. For example, a simple dominance argument eliminates all of the equilibrium points except one: $(7,\ldots,7)$. Any strategic uncertainty would cause an individually rational player to choose the payoff-dominant action, 7.

Table 3 reports the experimental results for treatment B and treatment A'. In period eleven, the payoff-dominant action, 7, was chosen by 84 percent of the subjects (76 of 91). However, the minimum in period eleven was never more than 4 and in experiments four, five, six, and seven it was a 1.[10]

Of course, a subject that adopts action 7 need not worry about what actions other subjects take and, apparently, most subjects did not. This property of dominating strategies resulted in the B treatment exhibiting different dynamics than the A treatment. Like the A treatment, those players who determine the minimum increase their action, but, unlike the A treatment, those players who were above the minimum do not decrease their action.[11] This dynamic behavior converges to the efficient outcome—the payoff-dominant equilibrium—in four of the six experiments. By period fifteen, 96 percent of the subjects chose the payoff-dominant action, 7.

Even in the experiments that obtained the efficient outcome, the B treatment was not sufficient to induce the groups to implement the payoff-dominant equilibrium in treatment A': parameter b equals \$0.10 once again. Returning to the original payoff table in period sixteen, 25 percent of the subjects chose the payoff-dominant action, 7.[12] However, 37 percent chose the secure action, 1. Period sixteen predictions were peaked with subjects choosing a 7 predicting most subjects would choose 7 and subjects choosing a 1 predicting most subjects would choose 1. This bi-modal distribution of actions and predictions suggests that play prior to period sixteen influenced subjects' behavior. However, the subjects exhibit a heterogenous response to this history.

Security predicts the stable outcome of treatment A'. In treatment A', the minimum in all periods of all six experiments was 1. By period twenty, 84 percent of the subjects chose the secure action, 1, and 94 percent chose an action less than or equal to 2. (Experiments two and four even satisfy the

[10]At least one subject did not understand how the payoff table had changed. Subject 3 in experiment 5, who plays a 1 in every period of the B treatment, predicts that all 16 players will choose 1 but only he does so. When the actual distribution was revealed, subject 3 appeared genuinely amazed and confessed that he had not understood how the payoffs had changed.

[11]The two exceptions were due to subject 3 in experiment five, see fn. 10, and subject 12 in experiment six. Subject 12 predicts that he will uniquely determine the minimum in period 11, verifies this in period 12 by choosing a 3, and then chooses a one for the remainder of the B treatment. Perhaps, subject 12 became vindictive. He had chosen a 7 in period 1.

[12]The large fluctuations in behavior resulting from changes in the parameter b—between treatment A and B and between treatment B and A'—suggest that subjects are influenced by the description of the game. In our view, these data are inconsistent with backward-looking theories of adaptive behavior.

TABLE 3—EXPERIMENTAL RESULTS FOR TREATMENT *B* AND TREATMENT *A′*

	Treatment *B*					Treatment *A′*				
	11	12	13	14	15	16	17	18	19	20
Experiment 2										
No. of 7's	13	15	16	16	16	8	2	0	0	0
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	0	1	0	0	0	1	0	0	0	0
No. of 4's	1	0	0	0	0	1	2	0	0	0
No. of 3's	1	0	0	0	0	1	1	1	1	0
No. of 2's	0	0	0	0	0	3	3	4	2	0
No. of 1's	0	0	0	0	0	2	8	11	13	16
Minimum	3	5	7*	7*	7*	1	1	1	1	1*
Experiment 3										
No. of 7's	13	13	12	13	14	6	2	2	1	1
No. of 6's	0	0	1	1	0	1	0	0	0	0
No. of 5's	0	0	1	0	0	0	2	1	0	0
No. of 4's	1	0	0	0	0	1	0	0	0	1
No. of 3's	0	1	0	0	0	0	0	0	0	0
No. of 2's	0	0	0	0	0	2	4	2	3	0
No. of 1's	0	0	0	0	0	4	6	9	10	12
Minimum	4	3	5	6	7*	1	1	1	1	1
Experiment 4										
No. of 7's	12	13	14	14	15	3	1	0	0	0
No. of 6's	0	0	0	0	0	0	0	0	0	0
No. of 5's	1	0	0	1	0	0	0	0	0	0
No. of 4's	0	1	1	0	0	2	0	0	0	0
No. of 3's	0	1	0	0	0	2	0	0	0	0
No. of 2's	0	0	0	0	0	2	1	2	0	0
No. of 1's	2	0	0	0	0	6	13	13	15	15
Minimum	1	3	4	5	7*	1	1	1	1*	1*
Experiment 5										
No. of 7's	13	13	15	15	15	1	0	0	0	0
No. of 6's	0	0	0	0	0	0	0	0	0	0
No. of 5's	1	1	0	0	0	0	0	0	0	0
No. of 4's	1	1	0	0	0	0	0	0	0	0
No. of 3's	0	0	0	0	0	1	1	0	0	0
No. of 2's	0	0	0	0	0	3	4	2	2	3
No. of 1's	1	1	1	1	1	11	11	14	14	13
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 6										
No. of 7's	13	13	12	12	13	2	2	2	2	2
No. of 6's	0	1	1	1	0	0	0	0	0	0
No. of 5's	0	1	1	0	1	0	0	0	0	0
No. of 4's	1	0	1	1	0	1	0	0	0	0
No. of 3's	0	1	0	1	0	1	0	0	0	0
No. of 2's	1	0	0	0	1	5	6	7	6	5
No. of 1's	1	0	1	1	1	7	8	7	8	9
Minimum	1	3	1	1	1	1	1	1	1	1
Experiment 7										
No. of 7's	12	14	13	13	14	3	4	2	2	2
No. of 6's	0	0	1	0	0	0	0	0	0	0
No. of 5's	0	0	0	0	0	1	0	0	0	0
No. of 4's	1	0	0	0	0	2	0	0	0	0
No. of 3's	0	0	0	1	0	2	0	0	0	0
No. of 2's	0	0	0	0	0	2	4	2	2	1
No. of 1's	1	0	0	0	0	4	6	10	10	11
Minimum	1	7*	6	3	7*	1	1	1	1	1

* ~ Denotes a mutual best-response outcome.

mutual best-response property of an equilibrium by period twenty.) Obtaining the efficient outcome in treatment B failed to reverse the observed coordination failure. Like the A treatment, the most inefficient outcome obtained.

V. Experimental Results for Treatment C: Group Size Two

Treatment C was added to the September experiments to determine if subjects were influenced by group size when choosing their actions. In theory, any uncertainty about the actions of an individual player in the game is exacerbated by the minimum rule as the number of players increases, see Section II. The C treatment reduces group size to two.

Table 4 reports the experimental results for the C treatment of experiments four and five, which permanently paired subjects with an unknown partner. In period twenty-one, 42 percent of the subjects play their payoff-dominant action, 7, and 74 percent of the subjects increase their action. This result occurred even though the minimum for the preceding five periods had been a 1 and all 31 subjects had played either a 1 (28 subjects) or a 2 (3 subjects) in period twenty. Clearly, either the subjects thought that their partner in treatment C would change his (her) action in response to reduced group size or the subjects expected alternative dynamics in repeated play.

The subjects in experiments four and five used an adaptive behavior in the C treatment similar to the adaptive behavior exhibited in the A treatment. Subjects that played the minimum increased effort by an average of $+2.0$ and subjects that played above the minimum reduced effort by an average of -1.9. However, unlike the A treatment, there was no "overshooting" to the secure action, 1. Occasionally, both subjects simultaneously chose the payoff-dominant action, 7.[13]

This dynamic behavior converged to the efficient outcome—the payoff-dominant equilibrium—in 12 of 14 trails. Hence, even with an extremely negative history payoff-dominance predicts the stable outcome of the tacit coordination game with fixed pairs.

Experiments six and seven randomly paired subjects with an unknown partner.[14] Hence, experiments six and seven test whether the results obtained in Experiments four and five were due to subjects repeating the period game with the same opponent. In the first period of these experiments, 37 percent of the subjects chose the payoff-dominant action, 7, and 73 percent of the subjects increased their choice of action, see Table 5. Moreover, the subjects' dynamic behavior was similar to that found in the fixed pair C treatment. While the results for the random pair C treatment are influenced by group size, no stable outcome obtains.

The C treatment confirms that there are two consequences of the minimum rule. First, group size interacting with the minimum rule alters the subjects' initial choice of action. Second, group size interacting with the minimum rule alters the convergence of the subjects' dynamic behavior in disequilibrium.

[13]Recall that subjects only observe the minimum and their own action. Hence, it is not possible to unilaterally "signal" a willingness to implement the payoff-dominant equilibrium, that is, subjects could not use Osborne's (1987) refinement of a "convincing deviation" to inform their behavior, see also van Damme (1987).

[14]Experiments by Cooper, DeJong, Forsythe, and Ross (1987) report that after eleven repetitions randomly paired groups of size two almost always obtain the payoff-dominant equilibrium. However, Cooper et al. also report coordination failure when subjects can choose from a strategy space that includes certain kinds of dominated cooperative strategies. Their game illustrates an interesting distinction between Luce and Raiffa's "solution in the strict sense," which depends on joint-admissibility and Harsanyi and Selten's solution, which depends on payoff-dominance. Because the first-best outcome requires using a strictly dominated strategy—as in the prisoner's dilemma game—and because joint-admissibility admits efficiency comparisons with disequilibrium outcomes, the Cooper et al. game with dominated cooperative strategies has no "solution in the strict sense" of Luce and Raiffa. Because the first-best outcome is an equilibrium in period game A, joint-admissibility—appropriately defined for n person games—and payoff-dominance select the same equilibrium point in period game A.

TABLE 4—EXPERIMENTAL RESULTS FOR TREATMENT *C*: FIXED PAIRINGS

	Period						
	21	22	23	24	25	26	27
Experiment 5							
Pair 1							
Subject 1	7	7	7	7	7	7	7
Subject 16	7	7	7	7	7	7	7
Minimum	7*	7*	7*	7*	7*	7*	7*
Pair 2							
Subject 2	7	2	7	7	7	7	7
Subject 15	1	7	3	6	7	7	7
Minimum	1	2	7	7	7	7	7
Pair 3							
Subject 3	1	1	1	1	1	1	1
Subject 14	1	1	7	1	1	1	7
Minimum	1*	1*	1	1*	1*	1*	1
Pair 4							
Subject 4	1	7	7	7	7	7	7
Subject 13	7	2	5	7	7	7	7
Minimum	1	2	5	7*	7*	7*	7*
Pair 5							
Subject 5	1	7	4	7	7	7	7
Subject 12	1	4	7	7	7	7	7
Minimum	1	4	4	7*	7*	7*	7*
Pair 6							
Subject 6	5	7	7	7	7	7	7
Subject 11	7	7	7	7	7	7	7
Minimum	5	7*	7*	7*	7*	7*	7*
Pair 7							
Subject 7	1	7	6	7	7	7	7
Subject 10	5	3	6	7	7	7	7
Minimum	1	3	6*	7*	7*	7*	7*
Pair 8							
Subject 8	7	6	6	7	7	7	7
Subject 9	3	5	7	7	7	7	7
Minimum	3	5	6	7*	7*	7*	7*
Experiment 6							
Pair 1							
Subject 2	7	7	4	5	6	6	7
Subject 15	2	3	6	6	7	7	7
Minimum	2	3	4	5	6	6	7*
Pair 2							
Subject 3	5	7	7	7	7	7	7
Subject 14	7	7	7	7	7	7	7
Minimum	5	7*	7*	7*	7*	7*	7*
Pair 3							
Subject 4	1	1	1	1	4	4	1
Subject 13	7	1	1	3	1	1	2
Minimum	1	1*	1*	1	1	1	1
Pair 4							
Subject 5	5	7	7	7	7	7	7
Subject 12	7	7	7	7	7	7	7
Minimum	5	7*	7*	7*	7*	7*	7*

TABLE 4—FIXED PAIRINGS, Continued

	Period						
	21	22	23	24	25	26	27
Pair 5							
Subject 6	4	5	7	7	7	7	7
Subject 11	4	5	7	7	7	7	7
Minimum	4*	5*	7*	7*	7*	7*	7*
Pair 6							
Subject 7	5	7	7	7	7	7	7
Subject 10	5	7	7	7	7	7	7
Minimum	5*	7*	7*	7*	7*	7*	7*

* ~ Denotes a mutual best-response outcome.

TABLE 5—DISTRIBUTION OF ACTIONS FOR TREATMENT *C*: RANDOM PAIRINGS

	Period				
	21	22	23	24	25
Experiment 6					
No. of 7's	5	5	4	10	8
No. of 6's	0	1	3	0	0
No. of 5's	2	5	3	3	4
No. of 4's	3	1	1	1	1
No. of 3's	1	1	1	0	0
No. of 2's	1	1	2	2	2
No. of 1's	4	2	2	0	1
Experiment 7					
No. of 7's	–	–	6	5	5
No. of 6's	–	–	1	0	1
No. of 5's	–	–	0	3	0
No. of 4's	–	–	2	1	4
No. of 3's	–	–	2	0	0
No. of 2's	–	–	0	0	1
No. of 1's	–	–	3	5	3

VI. Treatment A with Monitoring

As a referee points out, a reasonable conjecture is that revealing the distribution of actions each period—in addition to the minimum—might influence the reported dynamics. For example, subjects could signal a willingness to coordinate on the payoff-dominant equilibrium and optimistic subjects might delay reducing their action if they knew the minimum was determined by just one subject. Two experiments, each using payoff Table *A* and 16 naive subjects, were conducted in which the entire distribution of actions was recorded on a blackboard at the end of each period and was left there for the entire experiment.

The initial distribution of actions and the dynamics of the two monitoring experiments were similar to those reported above, see Table 6. If anything, the convergence of actions to the secure action, 1, was more rapid under the monitoring treatment. In fact, a mutual best-response outcome was obtained in one experiment: without monitoring mutual best-response outcomes are not observed for period game *A* until treatment *A*′. Apparently, monitoring helps solve the individual coordination problem—more subjects give a best-response sooner—but

TABLE 6—DISTRIBUTION OF ACTIONS FOR TREATMENT *A* WITH MONITORING

	Period									
	1^p	2^p	3	4	5	6	7	8	9	10^p
Experiment 8										
No. of 7's	4	0	0	0	2	0	0	1	1	1
No. of 6's	1	1	0	0	0	0	0	0	0	0
No. of 5's	4	0	1	0	0	1	0	0	0	1
No. of 4's	5	4	2	1	0	0	1	0	0	0
No. of 3's	1	4	1	0	0	2	0	0	0	0
No. of 2's	0	2	1	2	3	2	1	1	2	1
No. of 1's	1	5	11	13	11	11	14	14	13	13
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 9										
No. of 7's	6	2	0	1	0	0	0	0	0	0
No. of 6's	1	2	0	0	0	0	0	0	0	0
No. of 5's	2	2	1	0	0	0	0	1	0	0
No. of 4's	4	2	3	1	0	0	0	0	0	1
No. of 3's	1	5	3	1	0	0	0	0	1	0
No. of 2's	0	1	0	5	4	1	0	0	1	2
No. of 1's	2	2	9	8	12	15	16	15	14	13
Minimum	1	1	1	1	1	1	1*	1	1	1

^{p} ~ Denotes a period in which subjects made predictions.
* ~ Denotes a mutual best-response outcome.

not the collective coordination problem—the minimum was a 1 in all ten periods of both experiments.

VII. Concluding Comments

These experiments provide an interesting example of coordination failure. The minimum was never above four in period one and all seven experiments converged to a minimum of one within four periods. Since the payoff-dominant equilibrium would have paid all subjects $19.50 in the *A* and *A′* treatments—excluding predictions—and the average earnings were only $8.80, the observed behavior cost the average subject $10.70 in lost earnings.

This inefficient outcome is not due to conflicting objectives as in "prisoner's dilemma" games or to asymmetric information as in "moral hazard" games. Rather, coordination failure results from *strategic uncertainty*: some subjects conclude that it is too "risky" to choose the payoff-dominant action and most subjects focus on outcomes in earlier period games. The minimum rule interacting with this dynamic behavior causes the *A* and *A′* treatments to converge to the most inefficient outcome.

Deductive methods imply that all feasible actions are consistent with some equilibrium point in this experimental coordination game. However, the experimental results suggest that the first-best outcome, which is the payoff-dominant equilibrium, is an extremely unlikely outcome either initially or in repeated play. Instead, the results suggest that the initial outcome will not be an equilibrium point and only the secure—but very inefficient—equilibrium describes behavior that actual subjects are likely to coordinate on in repeated play of period game *A* when the number of players is not small.

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