Confidence Intervals

- Consider a sample $(X_1, X_2, ..., X_n)$ of some random variable
- Under the null hypothesis that $E(X_i) = \mu$, $\frac{\bar{X}-\mu}{se}$ has a t distribution with $n-1$ degrees of freedom
- This means

$$Prob\left(-t_{n-1,\alpha/2} \leq \frac{\bar{X}-\mu}{se} \leq t_{n-1,\alpha/2}\right) = 1-\alpha,$$

  where $t_{n-1,\alpha/2}$ is the critical value for an α-level hypothesis test
- Another way of stating this is:

$$Prob\left(\bar{X} - se \cdot t_{n-1,\alpha/2} \leq \mu \leq \bar{X} + se \cdot t_{n-1,\alpha/2}\right) = 1-\alpha$$

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$$Prob\left(\bar{X} - se \cdot t_{n-1,\alpha/2} \leq \mu \leq \bar{X} + se \cdot t_{n-1,\alpha/2}\right) = 1 - \alpha$$

- This means that if we drew the sample $(X_1, X_2, ..., X_n)$ over and over again (for example, by collecting new data), the true mean μ would lie within the interval

  $$[\bar{X} - se \cdot t_{n-1,\alpha/2}, \bar{X} + se \cdot t_{n-1,\alpha/2}]$$

  $100 * (1 - \alpha)\%$ of the time
- We call this the $100 * (1 - \alpha)\%$ **confidence interval**

Confidence Intervals

- 95% confidence interval for μ:
  $[\bar{X} - se \cdot t_{n-1,0.025}, \bar{X} + se \cdot t_{n-1,0.025}]$
- 99% confidence interval for μ:
  $[\bar{X} - se \cdot t_{n-1,0.005}, \bar{X} + se \cdot t_{n-1,0.005}]$
- **Exercise:** Consider the sample $X_1 = 1$, $X_2 = 2$ that we have been using before. Recall that the standard error of the mean in this sample is equal to 0.5. Use this information to construct 95% and 99% confidence intervals for μ.