Standard Error of the Mean

- Consider a sample $X_1, X_2, X_3, \ldots$ from some population with variance σ^2

- Recall that our estimate of the variance is given by

$$\text{Sample Variance} = S^2 = \frac{1}{N-1} \sum_n (X_n - \bar{X})^2$$

- So that our estimate of the standard deviation is

$$\sqrt{\frac{1}{N-1} \sum_n (X_n - \bar{X})^2}$$

- We now consider the problem of estimating the standard deviation of $\bar{X}$

- This object has a name: **standard error of the mean**

Standard Error of the Mean

We first need to remember that if X and Y are random variables, then $V(aX + bY) = a^2V(X) + b^2V(X)$. Now,

$$\begin{aligned} V(\bar{X}) = V\left(\frac{1}{N}\sum X_i\right) = V\left(\sum \frac{1}{N}X_i\right) = \\ = V(\frac{1}{N}X_1 + \frac{1}{N}X_2 + \frac{1}{N}X_3 + ...) = \\ = \frac{1}{N^2}\sigma^2 + \frac{1}{N^2}\sigma^2 + \frac{1}{N^2}\sigma^2 + ... \\ = \frac{\sigma^2}{N} \end{aligned}$$

Therefore, the standard deviation of $\bar{X}$ is equal to $\frac{\sigma}{\sqrt{N}}$

Standard Error of the Mean

- The standard deviation of $\bar{X}$ is equal to

$$\frac{\sigma}{\sqrt{N}}$$

- The **standard error of the mean** is the estimate of this subject
- Therefore, it is equal to $\frac{\hat{\sigma}}{\sqrt{N}}$, where $\hat{\sigma} = \sqrt{\frac{1}{N-1}\sum_n (X_n - \bar{X})^2}$

Example

- Assume that your sample of hourly wages (in dollars) is 20, 7, 13, and 8
- Compute the sample mean, sample variance, sample standard deviation, and the standard error of the mean