Beliefs

September 22, 2020

A survey

https://tinyurl.com/yypapkg7

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- $$PPV = \frac{\text{Number of sick people that would test positive in a random sample}}{\text{Total number of people that would test positive in a random sample}}$$

- Just like you would estimate the base rate as $\frac{\text{Number of sick people in a random sample}}{\text{Total number of people in a random sample}}$

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4. Estimate the number of healthy people that would test positive = $800 \cdot \textit{false positive rate} = 80$

5. Apply the counting rule:
$$PPV = \frac{\text{Sick people that would test positive}}{\text{All people that would test positive}}$$
so:
$$PVV = \frac{190}{190 + 80} = 0.704$$

link

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- $$P(A|B) = \frac{P(B|A)P(A)N}{P(B|A)P(A)N + P(B|\neg A)(1 - P(A))N}$$

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Bayes' rule is a mathematical fact

$$
\begin{aligned}
P(A|B) &= \frac{P(B \cap A)}{P(B)} \\
&= \frac{P(B|A)P(A)}{P(B)} \\
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\end{aligned}
$$

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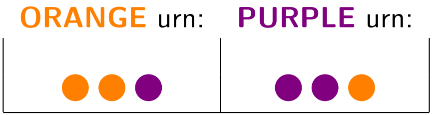
Terminology: $P(A)$=**prior**, $P(A|B)$=**posterior**

Urn example



- In the experiment you were asked to estimate the probability that an orange urn was used, $P($**ORANGE**$)$

- Prior to observing any draws from an urn, prior$=P($**Orange urn**$)=1/2$

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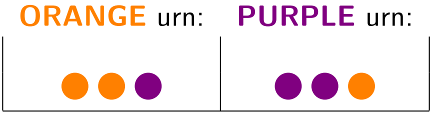

- In the experiment you were asked to estimate the probability that an orange urn was used, $P($**ORANGE**$)$
- Prior to observing any draws from an urn, prior$=P($**Orange urn**$) = 1/2$
- Consider a situation where the period 1 draw is **Orange**. What is the Bayesian posterior?

Urn example

ORANGE urn: **PURPLE** urn:

- $$P(\textbf{Orange urn}|\textbf{Orange ball}) = \frac{\text{Orange balls in the orange urn}}{\text{All orange balls}}$$
$$= 2/3$$

Getting to the same answer using Bayes' rule

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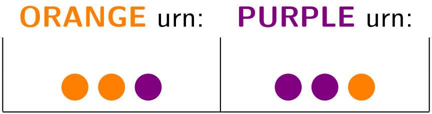
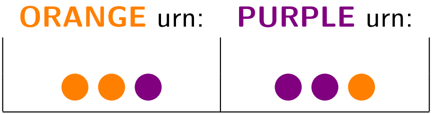
More draws/different priors



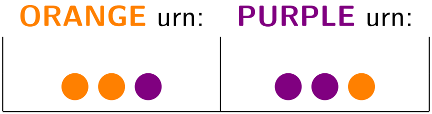
- Imagine now that one orange ball was observed in period 1
- The new prior is $P(\textbf{Orange urn}) = 2/3$

More draws/different priors



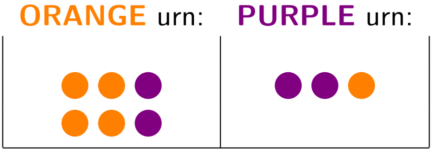
- Imagine now that one orange ball was observed in period 1
- The new prior is $P(\textbf{Orange urn}) = 2/3$
- Consider a situation where the period 2 draw is **Orange**. What is the new Bayesian posterior?

More draws/different priors



- We can represent the fact that an orange urn is twice as likely to be drawn by doubling the number of balls in the orange urn

More draws/different priors



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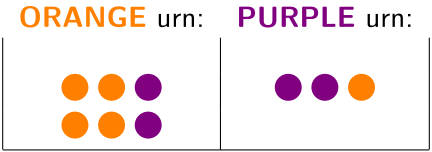
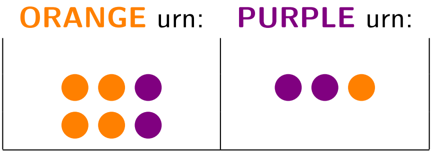


- Use the same intuition as before
- $P($**Orange urn**$|$**Orange ball**$) = 4/5$

More draws/different priors



- Use the same intuition as before
- $P(\textbf{Orange urn}|\textbf{Orange ball}) = 4/5$
- $P(\textbf{Orange urn}|\textbf{Purple ball}) = 1/2$

Using Bayes' rule

$$P(\text{Orange urn}|\text{Orange ball}) =$$

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$$= \frac{\frac{2}{3}P(\text{Orange urn})}{\frac{2}{3}P(\text{Orange urn}) + \frac{1}{3}P(\text{Purple urn})}$$

Using Bayes' rule

$$P(\textbf{Orange urn}|\textbf{Purple ball}) =$$

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Tom W. is a graduate student at the main university in your state. Please rank the following nine fields of graduate specialization in order of the likelihood that Tom W. is now a student in each of these fields. Use 1 for the most likely and 9 for the least likely:

- business administration
- computer science
- engineering
- humanities and education
- law
- library science
- medicine
- physical and life sciences
- social science and social work

The following is a personality sketch of Tom W. written during Tom's senior year in high school by a psychologist, on the basis of psychological tests of uncertain validity:

Tom W. is of high intelligence, although lacking in true creativity. He has a need for order and clarity, and for neat and tidy systems in which every detail finds its appropriate place. His writing is rather dull and mechanical, occasionally enlivened by somewhat corny puns and by flashes of imagination of the sci-fi type. He has a strong drive for competence. He seems to feel little sympathy for other people and does not enjoy interacting with others. Self-centered, he nonetheless has a deep moral sense.

Now please take a sheet of paper and rank the nine fields of specialization listen below by how similar the description of Tom W. is to the typical graduate student in each of the following fields. Use 1 for the most likely and 9 for the least likely:

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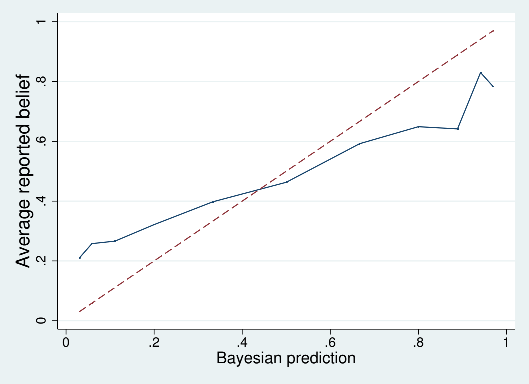
Base rate neglect

- Kahneman and Tversky (1973) find that subjects guess that Tom is most likely to be a computer scientist (high posterior probability)
- Even though computer science is a very unlikely major (low prior probability)
- $P(CS|Tom) = \frac{P(Tom|CS)P(CS)}{P(Tom)}$ and $P(CS) \approx 3\%$
- Subjects answer the simpler question about similarity instead of the more difficult question of probability (which they should be answering)

Our online experiment



Holt and Smith (2009

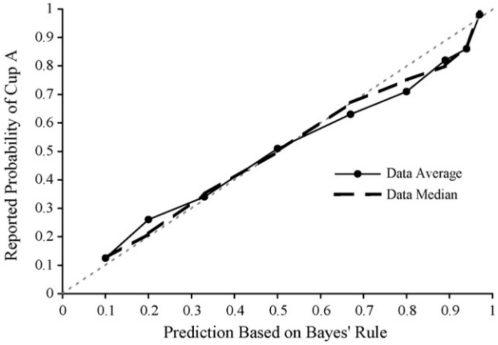
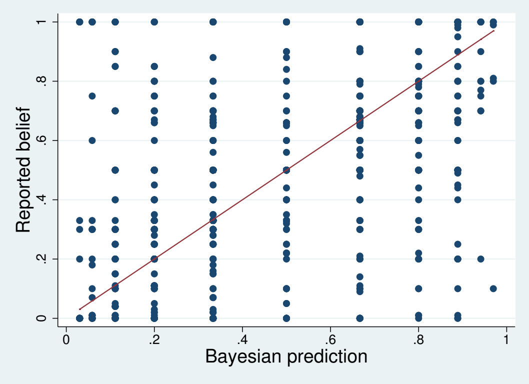


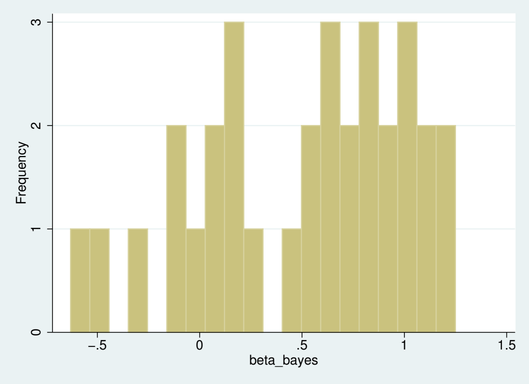
Fig. 1. Predictions versus average and median elicited probabilities for 22 subjects.

Our online experiment



Our online experiment

$belief_{it} = \beta_0 + \beta_1 bayes_{it} + \epsilon_{it}$, estimated at the individual level



Back to base rate neglect

- We can decompose how experimental subjects treat prior and new information following the approach of Grether (1980)
- Recall Bayes' rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)(1 - P(A))}$$

- Grether modifies the formula as follows:

$$P(A|B) = \frac{P(B|A)^{\alpha}P(A)^{\beta}}{P(B|A)^{\alpha}P(A)^{\beta} + P(B|\neg A)^{\alpha}(1 - P(A))^{\beta}}$$

$$P(A|B) = \frac{P(B|A)^{\alpha} P(A)^{\beta}}{P(B|A)^{\alpha} P(A)^{\beta} + P(B|\neg A)^{\alpha} (1 - P(A))^{\beta}}$$

$$\Leftrightarrow$$

$$\frac{P(A|B)}{P(\neg A|B)} = \left[\frac{P(B|A)}{P(B|\neg A)}\right]^{\alpha} \left[\frac{P(A)}{P(\neg A)}\right]^{\beta}$$

$$\Leftrightarrow$$

Log posterior odds = Log likelihood ratio of the signals · Log prior odds

- Let $P(\textbf{Orange urn}|s_t) = p_t$ denote the period-t posterior belief

- $s_t \in \{\textbf{Orange ball}, \textbf{Purple ball}\}$ is the period t signal

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- According to Grether's formula,

$$\frac{P(\textbf{Orange urn}|s_t)}{P(\textbf{Purple urn}|s_t)} = \left[\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})}\right]^{\alpha}\left[\frac{P(\textbf{Orange urn})}{P(\textbf{Purple urn})}\right]^{\beta}$$

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- Notice that the prior is the period-$t-1$ posterior

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- $s_t \in \{\textbf{Orange ball}, \textbf{Purple ball}\}$ is the period t signal

- According to Grether's formula,

$$\frac{p_t}{1-p_t} = \left[\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})}\right]^{\alpha}\left[\frac{p_{t-1}}{1-p_{t-1}}\right]^{\beta}$$

- Take logs of both sides and add an error term

$$log\left(\frac{p_t}{1-p_t}\right) = \alpha \cdot log\left[\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})}\right] + \beta \cdot log\left[\frac{p_{t-1}}{1-p_{t-1}}\right] + \epsilon_t$$

- **This is a regression we can estimate!**

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- **This is a regression we can estimate!**
- p_t is reported beliefs in period t

$$log\left(\frac{p_t}{1-p_t}\right) = \alpha \cdot log\left[\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})}\right] + \beta \cdot log\left[\frac{p_{t-1}}{1-p_{t-1}}\right] + \epsilon_t$$

- **This is a regression we can estimate!**
- p_t is reported beliefs in period t
- $\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})} = 2$ if the ball is orange
- $\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})} = 1/2$ if the ball is purple

$$log\left(\frac{p_t}{1-p_t}\right) = \alpha \cdot log\left[\frac{P(s_t|\textbf{Orange urn})}{P(s_t|\textbf{Purple urn})}\right] + \beta \cdot log\left[\frac{p_{t-1}}{1-p_{t-1}}\right] + \epsilon_t$$

- α captures how people respond to new information contained in the signal
- β captures how people respond to prior information
- Grether interprets $\alpha > \beta \geq 0$ as evidence of base rate neglect

Results from classroom experiment

```
. reg y x_signal x_prior if sum_empty==0, cluster(email)

Linear regression                                      Number of obs =     920
                                                       F(  2,     31) =   66.01
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.3568
                                                       Root MSE      =  1.9339

                                  (Std. Err. adjusted for 32 clusters in email)
------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    x_signal |     1.0056   .1893621     5.31   0.000     .6193932    1.391806
     x_prior |   .6054893   .0982161     6.16   0.000     .4051761    .8058024
       _cons |  -.1176603    .108206    -1.09   0.285     -.338348    .1030273
------------------------------------------------------------------------------
```

Compare to Holt and Smith (2009), who estimate $\alpha = 1.027$ and $\beta = .713$!!!!

Applying the Grether model to our survey results

The questions:

1. What are the chances in 100 that someone in your socioeconomic group in Moscow has COVID-19?
2. Consider a typical antibodies test for COVID-19. If someone with COVID-19 antibodies takes this test, what do you think are the chances in 100 that they would correctly test positive?
3. Consider a typical antibodies test for COVID-19. If someone without COVID-19 antibodies takes this test, what do you think are the chances in 100 that they would correctly test negative?
4. Imagine that a person in your socioeconomic group takes a typical antibodies test for COVID-19 and tests positive. What are the chances in 100 that they have the disease?
5. Imagine that a person in your socioeconomic group takes a typical antibodies test for COVID-19 and tests negative. What are the chances in 100 that they have the disease?

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4. Imagine that a person in your socioeconomic group takes a typical antibodies test for COVID-19 and tests positive. What are the chances in 100 that they have the disease?
5. Imagine that a person in your socioeconomic group takes a typical antibodies test for COVID-19 and tests negative. What are the chances in 100 that they have the disease?

Let $A =$COVID-19 present, $B =$positive test result

Applying the Grether model to our survey results

The questions:

1. What are the chances in 100 that someone in your socioeconomic group in Moscow has COVID-19? $P(A)$
2. Consider a typical antibodies test for COVID-19. If someone with COVID-19 antibodies takes this test, what do you think are the chances in 100 that they would correctly test positive?
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5. Imagine that a person in your socioeconomic group takes a typical antibodies test for COVID-19 and tests negative. What are the chances in 100 that they have the disease?

Let $A =$COVID-19 present, $B =$positive test result

Applying the Grether model to our survey results

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Let A =COVID-19 present, B =positive test result

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Let A =COVID-19 present, B =positive test result

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Let $A =$COVID-19 present, $B =$positive test result

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Let $A =$COVID-19 present, $B =$positive test result

Applying the Grether model to our survey results

- $A =$COVID-19 present
- $B =$positive test result

$$P(A|B) =$$

Applying the Grether model to our survey results

- A =COVID-19 present
- B =positive test result

$$P(A|B) = \frac{P(B|A)^{\alpha}P(A)^{\beta}}{P(B|A)^{\alpha}P(A)^{\beta} + P(B|\neg A)^{\alpha}P(\neg A)^{\beta}}$$

Applying the Grether model to our survey results

- A =COVID-19 present
- B =positive test result

$$P(A|B) = \frac{P(B|A)^\alpha P(A)^\beta}{P(B|A)^\alpha P(A)^\beta + P(B|\neg A)^\alpha P(\neg A)^\beta}$$

$$P(\neg A|B) =$$

Applying the Grether model to our survey results

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$$\frac{P(A|B)}{P(\neg A|B)} = \left[\frac{P(B|A)}{P(B|\neg A)}\right]^\alpha \left[\frac{P(A)}{P(\neg A)}\right]^\beta$$

Applying the Grether model to our survey results

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$$\frac{P(A|B)}{P(\neg A|B)} = \left[\frac{P(B|A)}{P(B|\neg A)}\right]^{\alpha}\left[\frac{P(A)}{P(\neg A)}\right]^{\beta}$$

$$log\left(\frac{P(A|B)}{P(\neg A|B)}\right) = \alpha \cdot log\left[\frac{P(B|A)}{P(B|\neg A)}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

- $A =$COVID-19 present
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$$log\left(\frac{R4}{P(\neg A|B)}\right) = \alpha \cdot log\left[\frac{P(B|A)}{P(B|\neg A)}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

- $A =$COVID-19 present
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$$\frac{P(A|B)}{P(\neg A|B)} = \left[\frac{P(B|A)}{P(B|\neg A)}\right]^\alpha \left[\frac{P(A)}{P(\neg A)}\right]^\beta$$

$$log\left(\frac{R4}{1-R4}\right) = \alpha \cdot log\left[\frac{P(B|A)}{P(B|\neg A)}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

- A =COVID-19 present
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$$P(A|B) = \frac{P(B|A)^{\alpha} P(A)^{\beta}}{P(B|A)^{\alpha} P(A)^{\beta} + P(B|\neg A)^{\alpha} P(\neg A)^{\beta}}$$

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$$\frac{P(A|B)}{P(\neg A|B)} = \left[\frac{P(B|A)}{P(B|\neg A)}\right]^{\alpha} \left[\frac{P(A)}{P(\neg A)}\right]^{\beta}$$

$$log\left(\frac{R4}{1 - R4}\right) = \alpha \cdot log\left[\frac{R2}{P(B|\neg A)}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

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$$log\left(\frac{R4}{1-R4}\right) = \alpha \cdot log\left[\frac{R2}{1-R3}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

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$$log\left(\frac{R4}{1-R4}\right) = \alpha \cdot log\left[\frac{R2}{1-R3}\right] + \beta \cdot log\left[\frac{R1}{1-R1}\right] + \epsilon$$

Applying the Grether model to our survey results

- $A =$COVID-19 present
- $B =$negative test result

$$P(A|B) = \frac{P(B|A)^{\alpha} P(A)^{\beta}}{P(B|A)^{\alpha} P(A)^{\beta} + P(B|\neg A)^{\alpha} P(\neg A)^{\beta}}$$

$$P(\neg A|B) = \frac{P(B|\neg A)^{\alpha} P(\neg A)^{\beta}}{P(B|\neg A)^{\alpha} P(\neg A)^{\beta} + P(B|A)^{\alpha} P(A)^{\beta}}$$

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$$log\left(\frac{R5}{P(\neg A|B)}\right) = \alpha \cdot log\left[\frac{P(B|A)}{P(B|\neg A)}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

- A =COVID-19 present
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$$P(A|B) = \frac{P(B|A)^{\alpha} P(A)^{\beta}}{P(B|A)^{\alpha} P(A)^{\beta} + P(B|\neg A)^{\alpha} P(\neg A)^{\beta}}$$

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$$log\left(\frac{R5}{1-R5}\right) = \alpha \cdot log\left[\frac{1-R2}{R3}\right] + \beta \cdot log\left[\frac{P(A)}{P(\neg A)}\right] + \epsilon$$

Applying the Grether model to our survey results

- $A =$COVID-19 present
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$$P(A|B) = \frac{P(B|A)^\alpha P(A)^\beta}{P(B|A)^\alpha P(A)^\beta + P(B|\neg A)^\alpha P(\neg A)^\beta}$$

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