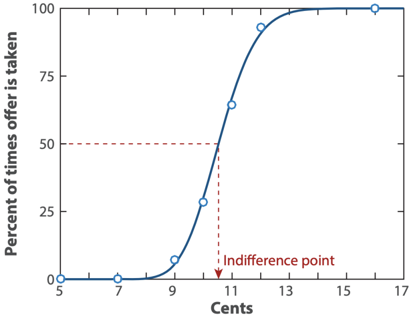
Stochastic Choice (Part 2)

October 25, 2021

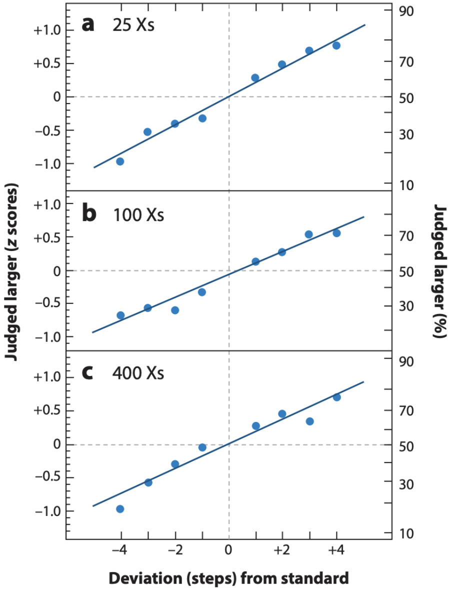
Risky choice

- Consider a choice between sure amount C and a lottery that pays X with probability p
- Standard approach assumes that subjects compare $C^{1-\sigma}$ to $0.5X^{1-\sigma}$, where σ is a risk aversion parameter
- Problem (choice of whether to give up 5 cents to obtain X with 50% probability):



Modeling imprecision in choice

- Models of stochastic choice typically take the form of random preferences
- $U(C) = C^{1-\sigma} + \epsilon$ with σ fixed
- $U(C) = C^{1-\sigma}$ with σ random
- It is still assumed that subjects maximize utility given the realization of the random element in preferences
- Woodford takes a different approach and assumed that the choice parameters themselves (C and X) are imperfectly observed
  - Assumptions about how C and X are coded inspired by the psychometrics literature



Number of x's in reference array fixed (x_1)

- Assume perception of steps in array i is recorded at $r_i \sim N(m(x_i), \sigma^2)$

$$
\begin{aligned}
P(r_1 > r_2) &= P(m(x_1) + \epsilon_1 > m(x_2) + \epsilon_2 \\
&= P(\epsilon_2 - \epsilon_1 < m(x_1) - m(x_2)) \\
&= P\left(\frac{\epsilon_2 - \epsilon_1}{\sqrt{2}\sigma} < \frac{m(x_1) - m(x_2)}{\sqrt{2}\sigma}\right) \\
&= \Phi\left(\frac{m(x_1) - m(x_2)}{\sqrt{2}\sigma}\right)
\end{aligned}
$$

$\Rightarrow$ z-score $\approx m'(x_1)(x_1 - x_2)\frac{1}{\sqrt{2}\sigma}$

Note **diminishing sensitivity** in previous figure; studies find that $m(x) = log(x)$ is a good fit to the data

Modeling imprecision in choice

- Consider again the choice between sure amount c and a lottery that pays x with probability p
- Assume c and x are imperfectly perceived
- $r_c \sim N(\log(c), \nu^2)$, $r_x \sim N(\log(x), \nu^2)$
- If $\log(x) \sim N(\mu, \sigma^2)$ and $\log(c) \sim N(\mu, \sigma^2)$, then $E(x|r_x) = exp((1-\beta)log(k) + \beta r_x)$ and $E(c|r_c) = exp((1-\beta)log(k) + \beta r_c)$, where $\beta = \frac{\sigma^2}{\sigma^2+\nu^2}$

$E(x|r_x) = exp((1-\beta)log(k) + \beta r_x)$ and
$E(c|r_c) = exp((1-\beta)log(k) + \beta r_c)$

$$E(c|r_c) < pE(x|r_x) \Leftrightarrow r_c - r_x < -\frac{1}{\beta}log(p^{-1})$$

$$\Leftrightarrow \frac{r_c - r_x - log(c/x)}{\sqrt{2}\nu} < \frac{-\frac{1}{\beta}log(p^{-1}) - log(c/x)}{\sqrt{2}\nu}$$

So, $P(\text{risky choice}) = \Phi\left(\frac{log(x/c) - \frac{1}{\beta}log(p^{-1})}{\sqrt{2}\nu}\right)$

Predictions

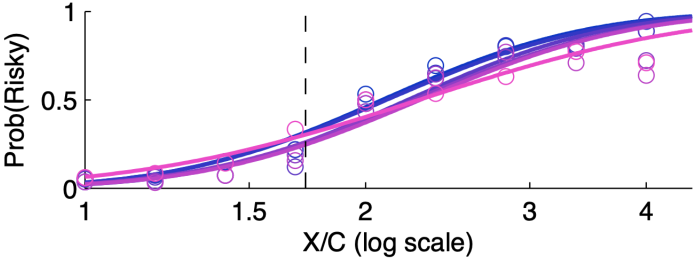
$$P(\text{risky choice}) = \Phi\left(\frac{log(x/c) - \frac{1}{\beta} log(p^{-1})}{\sqrt{2}\nu}\right)$$

- **Prediction 1**: Apparently risk-averse choices!
- **Prediction 2**: Proportion of risky choices is the same as long as ratio x/c is the same (scale invariance)
- **Prediction 3**: Greater randomness in perception implies more risk aversion

Experimental test

- 20 subjects with hundreds of trials for each subject
- $p = 0.58$ on all trials, C an X also not round numbers to prevent subjects from treating the problem as an arithmetic problem
- C takes one of: 5.55, 7.85, 11.1, 15,7, 22.2, or 31,4 (geometric series with each amount $\sqrt{2}$ larger than the previous one)
- $X = C \cdot 2^{m/4}$, where m is an integer between 0 and 8
- X/C takes on the same finite set of values for each value of C

Testing scale invariance



Discussion

- Not true that $\log(x) \sim N(\mu, \sigma^2)$ and $\log(c) \sim N(\mu, \sigma^2)$
- Scale invariance also holds for others models of random choice, e.g. random parameter model
- How much evidence is there for the connection between random choice and risk aversion?
- Seems to be no evidence in my recent experiment ($P = 0.4001$ correlation between consistent and risky choices)
- More tests of the model necessary. What if only one of c, x imperfectly perceived?