Prospect Theory

October 5, 2021

Expected utility model

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- Infinite expected value, but no one would pay very much for it
- Bernoulli (1738): gambles should be evaluated by their expected values:

$$EU = \sum u(x_i)p_i,$$

where $u(\cdot)$ is now called a von Nuemann-Morgenstern utility function

Attitudes to risk

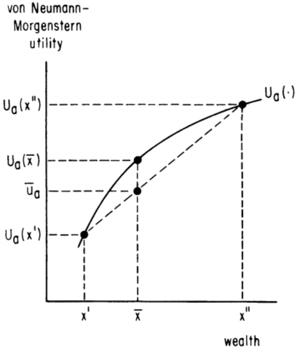

Fig. 1a. Concave utility function of a risk averter

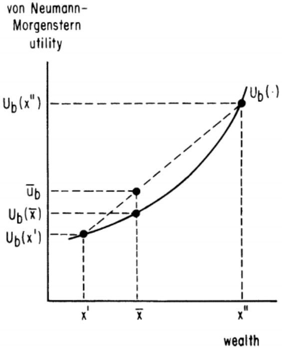

Fig. 1b. Convex utility function of a risk lover

A simple measure

- Fix a lottery, e.g., $(10, 0.5;\ 0, 0.5)$
- Let a subject choose between the lottery and a fixed alternative $p_t \in \{2, 3, ..., 7\}$.
- Suppose individual takes gamble at step t but fixed amount at step $t+1$
- Assume the individual would have been indifferent between the gamble and a 50/50 lottery giving p_t and p_{t+1}
- Assume $u(x) = x^{1-\sigma}$
- Solve for σ in

$$0.5u(10) + 0.5u(2) = 0.5u(p_t) + 0.5u(p_{t+1})$$

- Example of application: Burks, et al. (2009) study correlation between preferences and IQ

Holt and Laury (6000+ citations and counting)

Individual makes several decisions between a safe and a risky lottery, with same probability of getting the larger prize:

Decision	"Safe" pA	U(A)	1-pA	U(B)	EU	"Risky" pA	U(A)	1-pA	U(B)	EU
1	0.1	2	0.9	1.6	1.64	0.1	3.85	0.9	0.1	0.475
2	0.2	2	0.8	1.6	1.68	0.2	3.85	0.8	0.1	0.85
3	0.3	2	0.7	1.6	1.72	0.3	3.85	0.7	0.1	1.225
4	0.4	2	0.6	1.6	1.76	0.4	3.85	0.6	0.1	1.6
5	0.5	2	0.5	1.6	1.8	0.5	3.85	0.5	0.1	1.975
6	0.6	2	0.4	1.6	1.84	0.6	3.85	0.4	0.1	2.35
7	0.7	2	0.3	1.6	1.88	0.7	3.85	0.3	0.1	2.725
8	0.8	2	0.2	1.6	1.92	0.8	3.85	0.2	0.1	3.1
9	0.9	2	0.1	1.6	1.96	0.9	3.85	0.1	0.1	3.475
10	1	2	0	1.6	2	1	3.85	0	0.1	3.85

Under risk neutrality, should switch when EV of risky lottery is greater than that of a safe lottery

Switch point used to infer risk aversion

Rest of this class

ECONOMETRICA

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PROSPECT THEORY: AN ANALYSIS OF DECISION UNDER RISK

BY DANIEL KAHNEMAN AND AMOS TVERSKY[1]

What would you choose?

- Question 1
  - A: 80% chance of 4,000
  - B: 3,000 for sure
- Question 2
  - C: 20% chance of 4,000
  - D: 25% chance of 3,000

KT Results

Compare to:

PROBLEM 3:

A:	(4,000,.80),	or	B:	(3,000).
$N = 95$	[20]			[80]*

PROBLEM 4:

C:	(4,000,.20),	or	D:	(3,000,.25).
$N = 95$	[65]*			[35]

Allais Paradox

PROBLEM 3:

A: (4,000,.80), or B: (3,000).

$N = 95$ [20] [80]*

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).

$N = 95$ [65]* [35]

- The typical choice pattern (sometimes called the Allais Paradox) violates expected utility
- $u(3000) > 0.8u(4000) \Rightarrow 0.25u(3000) > 0.2u(4000)$
- Intuition: 3/4 chance of zero added to both lotteries and should be irrelevant

Allais Paradox

PROBLEM 3:

A: (4,000,.80), or B: (3,000).

$N = 95$ [20] [80]*

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).

$N = 95$ [65]* [35]

- Possible explanation: **probability weighting**
- $w(p) > p$ for small p and $w(p) < p$ for large p

Probability weighting

- In **expected utility theory**, $EU = \sum u(x_i)p_i$
- In **prospect theory**, $U = \sum u(x_i)w(p_i)$, where $w(\cdot)$ is some weighting function

Other examples of Allais paradox

- Problem A: 61% chance to win 520,000 or 63% chance to win $500,000
- Problem B: 98% chance to win 520,000 or 100% chance to win $500,000

“Certainty effect”

Weighting function in Kahneman and Tversky (1979)

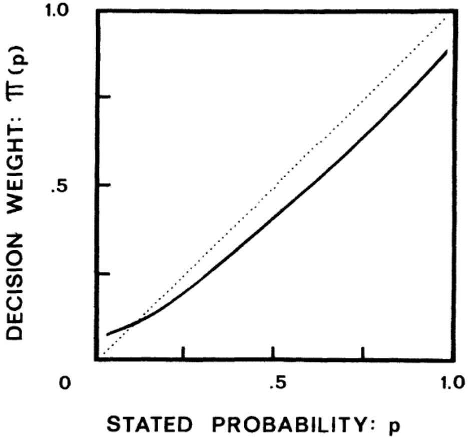

FIGURE 4.—A hypothetical weighting function.

What would you choose?

- Question 1
  - A: 25% chance of getting 6,000
  - B: 25% chance of getting 4,000, 25% chance of getting 2,000
- Question 2
  - A: 25% chance of getting -6,000
  - B: 25% chance of getting -4,000, 25% chance of getting -2,000

KT Results

PROBLEM 13:

	(6,000, .25),	or	(4,000, .25; 2,000, .25).
$N = 68$	[18]		[82]*

PROBLEM 13′:

	(−6,000, .25),	or	(−4,000, .25; −2,000, .25).
$N = 64$	[70]*		[30]

Reflection effect

These results suggest:

1. $u(6000) < u(4000) + u(2000)$
2. $u(-6000) > u(-4000) + u(-2000)$

Utility function is concave for gains and convex for losses!

(And therefore risk-averse for gains and risk-seeking for losses)

What would you choose?

- Question 1:
  In addition to whatever you own, you have been given 1,000.
  You are now asked to choose between:
  - A: 50% chance of getting 1,000
  - B: 500 for sure
- Question 2:
  In addition to whatever you own, you have been given 2,000.
  You are now asked to choose between:
  - A: 50% chance of -1,000
  - B: -500 for sure

KT results

Compare to:

PROBLEM 11: In addition to whatever you own, you have been given 1,000. You are now asked to choose between

A: (1,000, .50), and B: (500).

$N = 70$ [16] [84]*

PROBLEM 12: In addition to whatever you own, you have been given 2,000. You are now asked to choose between

C: (−1,000, .50), and D: (−500).

$N = 68$ [69*] [31]

Reference point

- These results suggest that utility is evaluated not over final wealth levels, but over gains and losses relative to a **reference point** (1000 in the first problem, 2000 in the second)
- Same problem stated differently leading to different answers $\Rightarrow$ **framing effect**

Another example

- Imagine that the United States is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows:
- If program A is adopted, 200 people will be saved
- If program B is adopted, there is a one-third probability that 600 people will be saved and a two-third probability that no people will be saved

Another example

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Others had the choices framed as follows:

- If program A' is adopted, 400 people will be die
- If program B' is adopted, there is a one-third probability that nobody people will die and a two-third probability that 600 people will die

Loss aversion

- The next piece of the puzzle is that most people would find the gamble $(x, .50; -x, .50)$ unattractive
- Utility of a loss is greater in absolute value than utility of an equivalent gain (**Loss aversion**)

Value function in Kahneman and Tversky (1979)

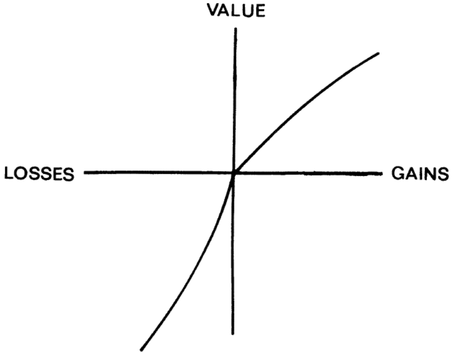


FIGURE 3.—A hypothetical value function.

Reference point
Loss aversion
Reflection effect
Diminishing marginal sensitivity

EUT vs. Prospect Theory

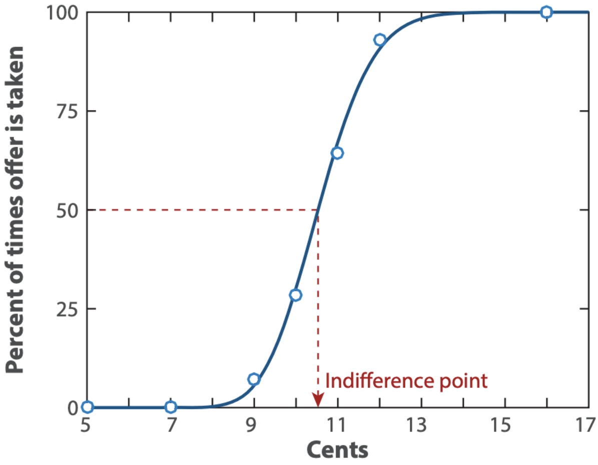
- What matters is final wealth vs. what matters is gains and losses around reference point
- No kink in utility vs. kink at reference point
- Constant attitudes to risk vs. risk-averse for gains and risk-seeking for losses
- Probabilities vs. probability weights

Prospect Theory: Some challenges

- What is the reference point? (Example: lottery vs. lottery)
- A prospect theoretic decision maker is still rational

Stochastic choice

Stochastic choice



Stochastic choice

- $U(Prospect) = EU(Prospect) + \epsilon$, ϵ i.i.d. Gumbel
- $U(c) = EU(c) + \epsilon$
- $U(L) = EU(L) + \epsilon$
- Then:

$$
\begin{aligned}
P(L) &= P(U(L) > U(C)) \\
&= P(EU(L) - EU(C) > \epsilon_c - \epsilon_L) \\
&= \frac{1}{1 + \exp(EU(c) - EU(L))}
\end{aligned}
$$

- (We can explain pattern of choice above)

Applications

- Stochastic choice can explain behavioral phenomena!
- Example: myopic loss aversion
- And perhaps even non-behavioral phenomena (e.g., risk-aversion in a lab as per Woodford)