Homework 8

1. Find the set of feasible, strictly individually rational payoffs in the following games:

	L	R
T	0,0	2,1
B	1,2	0,0

	L	R
T	1,-1	-1,1
B	-1,1	1,-1

	L	R
T	2,2	0,3
B	3,0	1,1

	L	R
T	1,1	0,0
M	1,1	2,1
B	0,0	2,1

2. Consider the following game:

	L	R
T	2,3	1,5
B	0,1	0,1

Assume that the discount factor is $\delta = 1/5$. Show that $((T, L), (T, L), ..)$ is not a subgame perfect equilibrium outcome path.

3. Consider the following prisoner's dilemma game:

	L_2	R_2
L_1	1,1	5,0
R_1	0,5	4,4

Assume that the game is infinitely repeated with discount factor γ. Consider the following strategy: Play R_i in the first period. In the t^{th} period, if the outcome of all preceding rounds has been (R_1, R_2), play R_i. Otherwise, play L_i. Show that this strategy is an equilibrium of the repeated game if $\gamma \geq 1/4$.

4. Consider a Cournot game with two firms producing q_1 and q_2 units of output,

with aggregate quantity in the market given by $Q = q_1 + q_2$. Assume that the market price is given by $P(Q) = a - Q$. Each firm has a marginal cost of c and no fixed costs.

- What is the Cournot (equilibrium) quantity? Cournot profit?
- What is the monopoly quantity? Monopoly profit?
- Consider the following trigger strategy: Produce half the monopoly quantity in the first period. In the t^{th} period, produce half the monopoly quantity if both firms have produced half the monopoly quantity in all preceding periods. Otherwise, produce the Cournot quantity. Show that it is a subgame perfect Nash equilibrium to play this strategy if $\delta \geq 9/17$.
- Now assume that the trigger strategy above produces the competitive quantity (instead of the Cournot quantity) in the punishment phase. What is the smallest discount factor for which this is a Nash Equilibrium? How does it compare to the discount factor in the previous part of the problem? Explain why it is larger/smaller.