Homework 6

Question 1 (Knowledge)

Let $I = \{1, 2\}$, $A^1 = \{U, D\}$, $A^2 = \{L, R\}$. The utilities are given below and assumed to be common knowledge:

	L	R
U	2,3	0,0
D	0,0	1,1

Suppose the state space is given by $\{\alpha, \beta, \gamma, \delta, \epsilon, \zeta\}$. Assume the following information partitions:

$$\mathcal{P}^1 = \{\{\alpha, \beta\}, \{\gamma, \delta\}, \{\epsilon, \zeta\}\}$$
$$\mathcal{P}^2 = \{\{\alpha, \gamma\}, \{\beta, \epsilon\}, \{\delta, \zeta\}\}$$

The actions are given by

- $\sigma^1(\alpha) = \sigma^1(\beta) = U$
- $\sigma^1(\gamma) = \sigma^1(\delta) = \sigma^1(\epsilon) = \sigma^1(\zeta) = D$
- $\sigma^2(\alpha) = \sigma^2(\gamma) = L$
- $\sigma^2(\beta) = \sigma^2(\epsilon) = \sigma^2(\delta) = \sigma^2(\zeta) = R$

The beliefs (conjectures) are given by

- $\mu^1(\alpha) = \mu^1(\beta) = \mu^1(\gamma) = \mu^1(\delta) = (\frac{1}{3}, \frac{2}{3})$
- $\mu^1(\epsilon) = \mu^1(\zeta) = (0, 1)$
- $\mu^2(\alpha) = \mu^2(\gamma) = \mu^2(\beta) = \mu^2(\epsilon) = (\frac{1}{4}, \frac{3}{4})$
- $\mu^2(\delta) = \mu^2(\zeta) = (0, 1)$

a) Are the conditions of Proposition 2 in Section 1.10 of the notes satisfied at α? What can you conclude about players' conjectures being a Nash Equilibrium? What do the players actually play at this state?

b) Are the conditions of Proposition 2 in Section 1.10 of the notes satisfied at ζ? What can you conclude about players' conjectures being a Nash Equilibrium? What do the players actually play at this state?

Question 2 (Dynamic Games)

Consider the following two player extensive form game. There are two players: Firm E (Entrant) and Firm I (Incumbent). Firm E chooses whether to enter a market or not. If it does not enter, the payoffs are 0 to Firm E and 2 to Firm I (Firm I gets all the profit). If Firm E enters, Firm I has two choices: to accommodate or fight. If Firm I accommodates, it gets a payoff of 1 while firm E gets a payoff of 2. If Firm I fights, Firm E gets a payoff of -3 and Firm I gets a payoff of -1.

(a) What are the Nash Equilibria?

(b) Argue that the Nash Equilibrium is not a sensible prediction for this game.