Homework 2

1. Consider a three player game with three firms $i = 1, 2, 3$. Each firm faces the demand curve $p = a - b(q_1 + q_2 + q_3)$ and per-unit costs of production c. Does iterated elimination of strictly domination actions yield a unique prediction in this game?

2. Prove formally that if $(R^1, ..., R^N)$ and $(T^1, ..., T^N)$ are rationalizable, then $(R^1 \cup T^1, ..., R^N \cup T^N)$ is rationalizable, as well.

3. (a) Argue that if a player has two weakly dominant strategies, then for every strategy choice of his opponents, the two strategies yield him equal payoffs. (b) Provide an example of a two player game in which a player has two weakly dominant strategies but his opponent prefers that he play one of them rather than the other.

4. Consider the following auction (known as a second-price, or Vickrey, auction). An object is auctioned off to N bidders. Bidder i's valuation of the object (in monetary terms) is v_i. The auction rules are that each player submit a bit (a nonnegative number) in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object but pays the auctioneer the amount of the *second-highest* bid. If more than one bidder submits the highest bid, each gets the object with equal probability. Show that submitting a bid of v_i with certainty is a weakly dominant strategy for bidder i.

5. Show that the set of mixed strategies S is nonempty, compact, and convex.

6. Show that for every s, $BR^i(s)$ is closed, convex, nonempty, and equal to the mixed strategies concentrated on $PBR^i(s)$.